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"THEORETICAL CONSIDERATION FOR THE MEASUREMENTS OF ATTENUATION OF MILLIMETRE AND CENTIMETRE WAVES IN THE RAIN FALL"

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I. Introduction

During the World War II, study of microwave region in U. S. A. made a rapid and remarkable progress through various and extensive researches. After the War, the results of measurements of attenuation of electromagnetic waves in the rain-fall region have been reported in succession: the results for 3.2*cm.* and 1.09*cm.* wave lengths by Robertson and King[1]) in April 1946, those for 1.25*cm.* wave length by Lloyd and Anderson[2]) in April 1947, and those for 0.62*cm.* wave length by Mueller[3]) in April 1946. In all cases the attenuation between the transmitter and the receiver about a hundred feet apart was measured in *db per mile*, and then the rain precipitation was also measured. These measurements are represented in Figures 1, 2, 3, and 4 by small circles.

Meanwhile theoretical researches related to this subject have also been made and propounded: computations for the colour of colloid by G. von Mie[4]) in 1908, theoretical contributions for the dielectric constant of water by P. Debye[5]) in 1927 and a research for the attenuation of electromagnetic waves in cloud and fog by K. Fränz[6]) in 1940. G. von Mie, solving Maxwell's equations exactly for the case where there is a dielectric sphere of arbitrary dielectric constant in a plane wave field, discussed the phenomena of scattering and absorption of light by the dilute colloidal dispersive medium. In this case it was assumed that the effect of the number of particles be equal to that of one particle multiplied by the number of particles

The ratio of the dimension of rain drops as dispersed particles to centimetre waves, in comparable to that of colloidal particles to visible rays; hence Mie's theory is applicable to our present study.

Debye's paper has discussed the dielectric constant and other material constants of a liquid composed of dipole molecules and how it changes as the frequency varies, and deduced the well-known Debye's Formulae.

Fränz has computed the attenuation of short waves in cloud and fog, on the basis of the computation of G. von Mie, with the dielectric constant of water obtained from Debye's theory of molecular dispersion. The fact that Fränz has worked on the cloud or fog instead of rain drops, means that the diameter of water drops is far smaller than the wave length of electro-magnetic waves and that he could take up only the first term of power series of *diameter/wave length*. Our case is of rain drops, and in the millimetre and centimetre region the drop size is of the same order to the wave length, so that, if we assume Rayleigh scattering after Fränz, the theory is contradictory to the observation.

To discuss the comparison of theory and experiments, we must compute theoretical values going back to Mie's paper. Further since only attenuation and precipitation are measured in the experiments, we must obtain the concentration of rain drops from precipitation by assuming drop size or falling speed, because it is only drop concentration in the wave path that is essential in the theoretical treatment. We have assumed drop size on the ground of several data, and since there is a relation between drop size and rain-fall velocity, we have been able to estimate from rain precipitation concentration of rain drops suspended in air.

It is the substance of this article to compare the theory of absorption and scattering of electromagnetic waves by rain-fall, thus obtained, with the experiments.

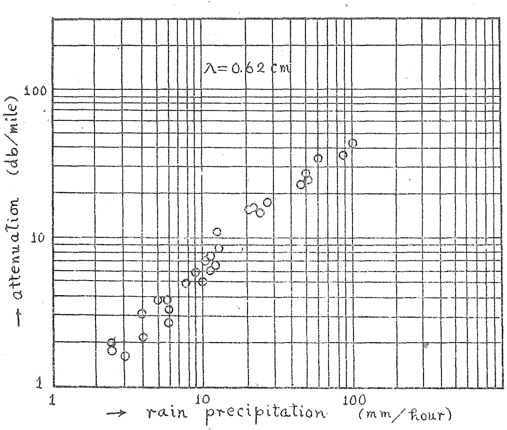


Fig. 1.

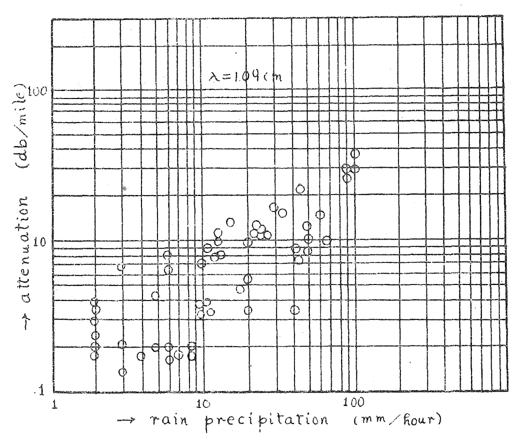


Fig. 2.

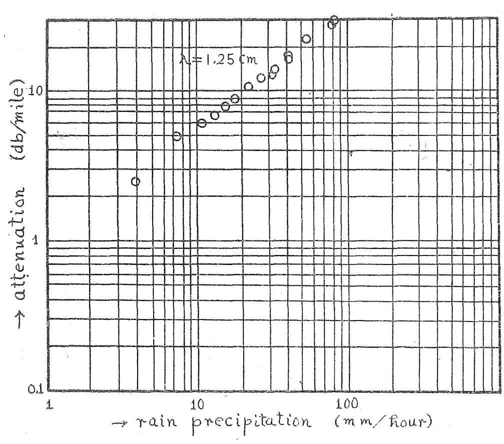


Fig. 3.

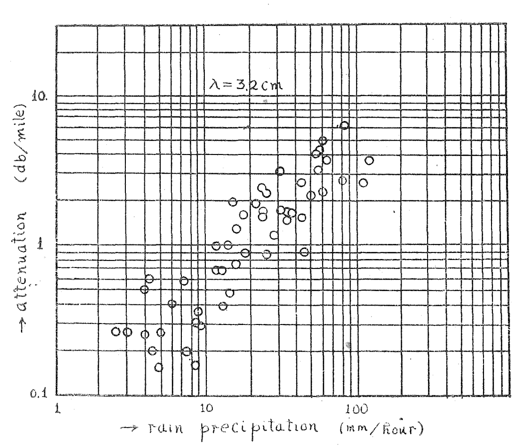


Fig. 4.

II. Calculation of attenuation coefficient.

To compute the attenuation theoretically, we must begin with the calculation of attenuation coefficient. After general and exact calculations, Mie gave the following formula as the absorption coefficient of colloidal solution;

$$k=N\frac{\lambda^2}{2\pi}Im\left\{\sum_{\nu-1}^{\infty}(-1)^{\nu}\ (a-p)\right\}, \tag{1}$$

where N is the number of particles *per* cm^3, λ is the wave length of electromagnetic waves in cm, and Im { } represents the imaginary part of { }. The Equ. (1) represents total attenuation in which are involved absorption and scattering. Herein attenuation due to scattering only is given by

$$k'=N\frac{\lambda^2}{2\pi}\sum_{\nu=1}^{\infty}\frac{|a_\nu|^2+|p_\nu|^2}{2\nu+1}. \tag{2}$$

In Equs. (1) and (2) k, k' are attenuation coefficients in the *naper per cm*, a_ν, and p_ν are relative amplitudes of electromagnetic waves in a particle to that of incident electromagnetic waves, which correspond to the coefficients of expansion of electromagnetic field after surface spherical harmonics, having two sorts of terms a's and p's respectively, as a result of expression the field as a sum of fields having only either electric or magnetic radial component. We call a_1, a_2,........; p_1, p_2, ... as electric, dipole quadrupole, ...and magnetic dipole, quadrupole, ... respectively, after Mie.

These a_ν and p_ν are given by

$$a_\nu=(2\nu+1)\cdot i_\nu\frac{I_\nu'(\alpha)\cdot I_\nu(\beta)\cdot\beta-I_\nu'(\beta)\cdot I_\nu(\alpha)\cdot\alpha}{K_\nu'(-\alpha)\cdot I_\nu(\beta)\cdot\beta-I_\nu'(\beta)\cdot K_\nu(-\alpha)\cdot\alpha} \tag{3}$$

$$p_\nu=-(2\nu+1)i_\nu\frac{I_\nu(\alpha)\cdot I_\nu'(\beta)\cdot\beta-I_\nu(\beta)\cdot I_\nu'(\alpha)\cdot\alpha}{K_\nu(-\alpha)\cdot I_\nu'(\beta)\cdot\beta-I_\nu(\beta)\cdot K_\nu'(-\alpha)\cdot\alpha} \tag{4}$$

here,

$$\alpha=\frac{2\pi\rho}{\lambda}, \tag{5}$$

$$\beta=\frac{2\pi\rho}{\lambda}n, \tag{6}$$

$$n^2=\varepsilon'-i\left(\varepsilon''+\frac{4\pi\sigma}{\omega}\right), \tag{7}$$

and ρ is the radius of rain drop, λ the wave length in cm, n the complex refractive index of water, and therefore a_ν, p_ν are represented as function of ρ/λ and n.

I_ν and K_ν are functions deduced from Bessel functions of half and odd integer orders, and I_ν', K_ν' are their first derivatives

$$I_\nu(x)=\sqrt{\frac{\pi}{2x}}\cdot J_{\nu+\frac{1}{2}}(x), \tag{8}$$

$$K_\nu(x) = i^{\nu+1}\sqrt{\frac{\pi}{2x}}\,H^{(1)}_{\nu+\frac{1}{2}}(x). \tag{9}$$

Concrete forms and various expanded forms of these functions are given in Mie's paper, but it is too laborious to give precise numerical evaluations of a_ν, p_ν.

But a_ν, p_ν can also be represented, using power series of α, β; u_ν, v_ν, w_ν, which have unity as initial terms, as follows,

$$a_\nu = (-1)^{\nu-1}\cdot\frac{\nu+1}{\nu}\cdot\frac{\alpha^{2\nu+1}}{1^2\cdot 3^2\cdots(2\nu-1)^2}\cdot u_\nu\cdot\frac{n^2-v_\nu}{n^2+\frac{\nu+1}{\nu}w_\nu}\cdot e^{i\alpha}, \tag{10}$$

$$p_\nu = (-1)^{\nu}\cdot\frac{\nu+1}{\nu}\cdot\frac{\alpha^{2\nu+1}}{1^2\cdot 3^2\cdots(2\nu-1)^2}\cdot u_\nu\cdot\frac{1-v_\nu}{1+\frac{\nu+1}{\nu}w_\nu}\cdot e^{i\alpha}, \tag{11}$$

For the values of α and β much smaller than unity, the above two expressions reduce to their first terms whose orders are given by $\alpha^{2\nu+1}$ and $\alpha^{2\nu+3}$, respectively. When α and β get larger, the other terms, term by term from their lower modes, are to be taken into consideration. If we put u_1, v_1, w_1, equal unity, for electric dipole, it becomes,

$$a_1 = 2\alpha^3\cdot e^{i\alpha}\cdot\frac{n^2-1}{n^2+2}\cdot \tag{12}$$

Thus it reduces to the so-called Rayleigh's scattering formula.

However, for the case under consideration, since we can not regard as $\alpha \ll 1$, $\beta \ll 1$, higher order terms of expansion of a_1 as well as higher mode terms of Equs. (1) and (2) are essential.

Therefore, we intended, returning to the Equs. (3) and (4), to calculate exactly electric dipole a_1, magnetic dipole p_1, electric quadrupole a_2 and magnetic quadrupole p_2. At this calculation we had to employ the following original function forms, since the abovementioned expanded formulae converge very slowly or diverge, α and β being comparable or greater than unit in our case.

$$\left.\begin{aligned}
I_1(x) &= -\cos x+\frac{\sin x}{x}\\
I_2(x) &= -\sin x-\frac{3\cos x}{x}+\frac{3\sin x}{x^2}\\
I'_1(x) &= \sin x+\frac{\cos x}{x}-\frac{\sin x}{x^2}\\
I'_2(x) &= -\cos x+\frac{3\sin x}{x}+\frac{6\cos x}{x^2}-\frac{6\sin x}{x^3}\\
K_1(-x) &= -\frac{i}{x}\cdot e^{-ix}\cdot(1+ix)\\
K_2(-x) &= -\frac{3}{x^2}\cdot e^{-ix}\cdot\left\{\left(1-\frac{1}{3}x^2\right)+ix\right\}\\
K'_1(-x) &= +\frac{i}{x^2}\cdot e^{-ix}\cdot\left\{(1-x^2)+ix\right\}
\end{aligned}\right\} \tag{13}$$

$$K_2(-x)=+\frac{6}{x^3}\cdot e^{-ix}\cdot\left\{(1-\frac{1}{2}x^2)+ix(1-\frac{1}{6}x^2)\right\}$$

And as for the value of refractive index n included in β, since it has a dispersing region around $2cm.$, it becomes naturally an imaginary number and varies with wave length. And in the imaginary part of n, $\frac{4\pi\sigma}{\omega}$ is far smaller than ε'' in our case of water and centimeter and millimeter wave region, so we can neglect $\frac{4\pi\sigma}{\omega}$ against ε''. Therefore, in our calculation, we employed dielectric constant and $tan\delta$ described in Fränz's paper as shown in Fig. 5, which are derived from the direct measurements by Esau and Bäz[7]) and others coincided with Debye's formula, and give the values at the temperature about 20°C. So we can calculate the values of ε and $tan\delta$ for every wave length from

$$n=\sqrt{\varepsilon}=\sqrt{\varepsilon'-i\varepsilon''}=\sqrt{\varepsilon'(1-itan\delta)}=\sqrt{|\varepsilon|}\,e^{-i\frac{\delta}{2}}$$

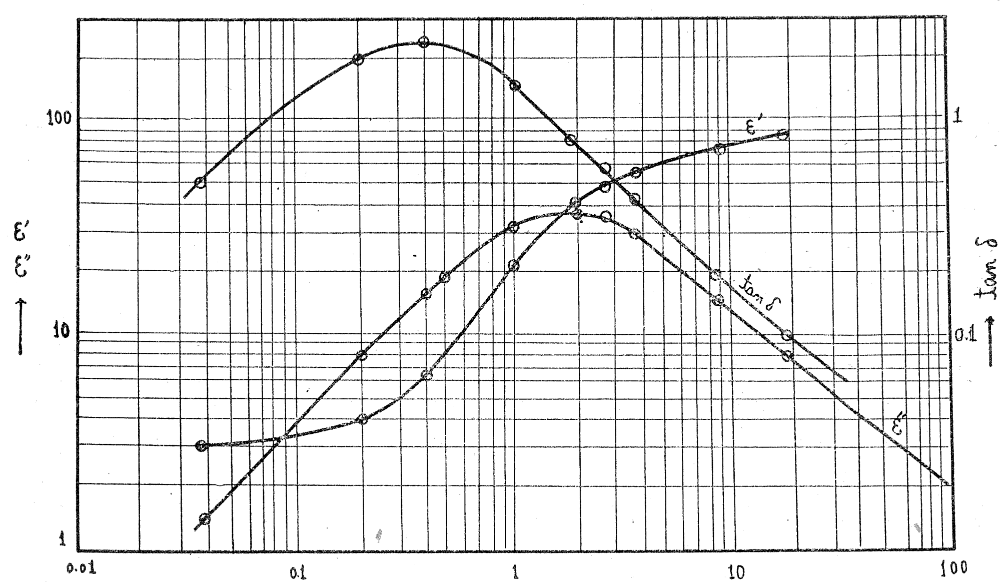


Fig5. ——wave length(cm)

The values of ε, $tan\delta$, a, and β, used in our calculation, are listed in Table 1.

$\lambda=0.62cm.$	$\lvert\varepsilon\rvert=25.05$	$tan\delta=1.956$				
drop dia 2ρ	$1mm$	$1.9mm$	$3.1mm$	$3.7mm$	$4mm$	$7mm$
a	0.507	0.962	1.57	1.87	2.03	3.55
$Re\{\beta\}$	2.16	4.11	6.70	8.00	8.65	15.1
$Im\{\beta\}$	1.32	2.51	4.10	4.90	5.29	9.26
$\lambda=1.09cm$	$\lvert\varepsilon\rvert=40.41$	$tan\delta=1.482$				

a	0.288	0.547	0.893	1.07	1.15	2.02
$Re\{\beta\}$	1.62	3.07	5.01	5.98	6.47	11.3
$Im\{\beta\}$	0.860	1.63	2.67	3.18	3.44	6.02
$\lambda=1.25cm$ $\lvert\varepsilon\rvert=44.56$ $tan\delta=1.360$						
a	0.251	0.477	0.779	0.930	1.01	1.76
$Re\{\beta\}$	1.50	2.84	4.64	5.54	5.99	10.5
$Im\{\beta\}$	0.757	1.44	2.35	2.80	3.03	5.29
$\lambda=3.2cm$ $\lvert\varepsilon\rvert=67.46$ $tan\delta=0.5542$						
a	0.0981	0.186	0.304	0.363	0.393	0.687
$Re\{\beta\}$	0.780	1.48	2.42	2.89	3.12	5.46
$Im\{\beta\}$	0.202	0.383	0.625	0.747	0.807	1.41

Table. 1.

With values given in Table 1 and from Equs. (13), (3), and (4), we could calculate $Im\{-a_1\}$, $Im\{p_1\}$, $Im\{a_2\}$, $Im\{-a_1+p_1+a_2\}$, and $|a_1|^2/3$, $|p_1|^2/3$, $|a_2|^2/5$, $\{|a_1|^2+p_1|^2\}/3+|a_2|^2/5$, for every observed wave length, and for 0.62cm wave $Im\{-p_2\}$ also as given in Table 2.

Wave length	Drop diameter 2ρ	1mm	1.9mm	3.1mm	3.7mm	4mm	7mm
0.62cm	$Im\{-a_1\}$	0.0639	1.12	1.67	1.45	1.27	1.10
	$Im\{p_1\}$	0.0522	0.253	0.832	1.32	1.59	1.94
	$Im\{a_2\}$	0.000662	0.0241	0.764	1.71	2.13	1.31
	$Im\{-p_2\}$	0.000910	0.0361	0.239	0.400	0.536	3.81
	$Im\{-a_1+p_1+a_2-p_2\}$	0.118	1.43	3.51	4.88	5.53	8.16
	$\lvert a_1\rvert^2/3$	0.0258	0.725	1.19	0.998	0.833	0.706
	$\lvert p_1\rvert^2/3$	0.000963	0.0637	0.552	1.02	1.28	1.59
	$\lvert a_2\rvert^2/5$	0.824×10^{-5}	0.00360	0.352	0.985	1.31	0.568
	$\lvert p_2\rvert^2/5$	1.88×10^{-7}	0.000397	0.0358	0.138	0.236	3.30
	$\{\lvert a_1\rvert^2+p_1\rvert^2\}/3+\{\lvert a_2\rvert^2+\lvert p_2\rvert^2\}/5$	0.0268	0.793	2.13	3.14	3.66	6.16
1.09cm	$Im\{-a_1\}$	0.00402	0.0835	0.846	1.31	1.47	1.18
	$Im\{p_1\}$	0.00586	0.0832	0.200	0.302	0.367	1.64
	$Im\{a_2\}$	0.416×10^{-4}	0.000569	0.0122	0.036	0.0727	1.71
	$Im\{-a_1+p_1+a_2\}$	0.00992	0.167	1.06	41.65	1.91	4.53
	$\lvert a_1\rvert^2/3$	0.000772	0.0421	0.560	0.931	1.07	0.805
	$\lvert p_1\rvert^2/3$	0.122×10^{-4}	0.000362	0.0505	0.120	0.172	1.38

	$\lvert a_2\rvert^2/5$	0.122×10^{-6}	0.136×10^{-4}	0.00172	0.00949	0.0245	1.10
	$\{\lvert a_1\rvert^2+\lvert p_1\rvert^2\}/3+\lvert a_2\rvert^2/5$	0.000784	0.0457	0.612	1.06	1.27	3.29
1.25*cm*	$Im\{-a_1\}$	0.00215	0.00397	0.502	0.938	1.15	1.47
	$Im\{p_1\}$	0.00313	0.0620	0.148	0.217	0.261	1.17
	$Im\{a_2\}$	0.35×10^{-5}	0.000217	0.0445	0.0153	0.0265	1.18
	$Im\{-a_1+p_1+a_2\}$	0.00528	0.102	0.654	1.17	1.44	3.82
	$\lvert a_1\rvert^2/3$	0.000329	0.0184	0.312	0.639	0.810	1.09
	$\lvert p_1\rvert^2/3$	0.361×10^{-5}	0.000163	0.0260	0.0638	0.0935	0.930
	$\lvert a_2\rvert^2/5$	0.335×10^{-8}	0.261×10^{-5}	0.000413	0.00247	0.00531	0.692
	$\{\lvert a_1\rvert^2--\lvert p_1\rvert^2\}/3+a\lvert_2\rvert^2/5$	0.000333	0.0201	0.338	0.705	0.909	2.71
3.2*cm*	$Im\{-a_1\}$	0.399×10^{-4}	0.000338	0.00261	0.00680	0.0108	0.243
	$Im\{p_1\}$	0.255×10^{-4}	0.000741	0.0180	0.0517	0.0629	0.113
	$Im\{a_2\}$	0.184×10^{-6}	0.0000	0.0000639	0.0000407	0.0000342	0.00210
	$Im\{-a_1+p_1+a_2\}$	0.565×10^{-4}	0.00108	0.0207	0.0585	0.737	0.358
	$\lvert a_1\rvert^2/3$	0.120×10^{-5}	0.547×10^{-4}	0.00108	0.00330	0.00532	0.148
	$\lvert p_1\rvert^2/3$	0.425×10^{-6}	0.554×10^{-6}	0.000134	0.000894	0.00159	0.0155
	$\lvert a_2\rvert^2/5$	0.375×10^{-11}	0.574×10^{-7}	0.535×10^{-6}	0.112×10^{-5}	0.227×10^{-5}	0.000132
	$\{\lvert a_1\rvert^2+\lvert p_1\rvert^2\}/3+\lvert a_2\rvert^2/5$	0.120×10^{-5}	0.553×10^{-4}	0.00121	0.00419	0.00691	0.163

Table 2.

III. Considerations on Rain Drop Size.

Exact estimation of raindrop size is a very hard problem to deal with, because it differs with rain character and, moreover, even in one rain fall it has a complicated distribution; therefore it becomes difficult to get proper conclusion in comparing theory and measurements. Fortunately, among papers on measurements, Lloyed and Anderson gave results of their efforts to get correlation between rain drop size and rain precipitation. This is shown in Fig. 6. According to this, mean drop diameters are mostly between 1 and 2*mm*. Here we have to pay attention to a statistical treatment in evaluation the mean drop diameter in one rain fall. Mean diameter is obtained from frequency distribution function, but frequency distribution function differs with independent variable employed. Frequency curve is defined by

$$f(x)=\frac{dn}{dx}, \tag{14}$$

when we take the number of measured value which fall in x to $x+dx$ as dn. Therefore, the frequency curve for the function of x ; $y=\phi(x)$ obtained from the relations

$$f(x)dx = g(y)dy = g\{\phi(x)\}\cdot\frac{dy}{dx}\cdot dx. \tag{15}$$

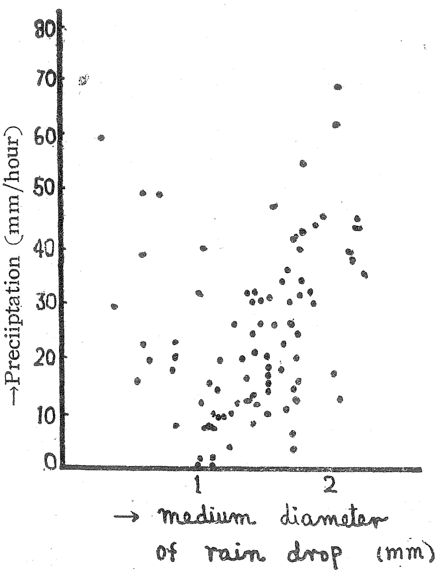


Fig. 6.

Therefore, to obtain the approximate mean diameter, there is a way to define a medium diameter x_0 gained from

$$\int_0^\infty f(x)dx = 2\int_0^{x_0} f(x)dx. \tag{16}$$

According to this, the medium value for other variable y amount to $y_0 = \phi(x_0)$ and for the case in which homogenous random value is unknown, we can get rid of the contradiction that mean values are different for the variables used. In the paper of Lloyed and Anderson medium diameter is employed according to this method. However. it is necessary to take a properly defined mean diameter applicable to the phenomena of scattering and absorption. Now, putting frequency curve for drop radius as $f(\rho)$, attenuation constant k, function of ρ as $k(\rho)$,

$$k(\rho) = CK(\rho)$$

$$C = N\frac{4}{3}\pi\rho^3 \quad \text{concentration } gr/cm^3$$

and effective mean radius is ρ_0 obtained from

$$\int_0^\infty K(\rho)\cdot\rho^3 f(\rho)d\rho = K(\rho_0)\rho_0^{\ 3}\int_0^\infty f(\rho)d\rho. \tag{17}$$

Strictly speaking, it can be gained only after determination of $K(\rho)$ curve. But, since the variation of $K(\rho)$ is small, ρ'_0 is obtained as the mean value for the frequency curve $\rho^3 f(\rho)d\rho$;

$$\int_0^\infty \rho^3 f(\rho)d\rho = 2\int_0^{\rho'_0} \rho^3 f(\rho)d\rho. \tag{17}$$

And we take medium value in place of this. We call this ρ'_0 as mass medium radius for convenience. So the ratio of this to the conventional medium radius

$$\frac{\rho'_0}{\rho_0} = \eta \tag{18}$$

is constant for the definite frequency distribution curve.

It is difficult to determine the form of frequency distributien curve, but authors obtained ρ'_0, and ρ_0 from two results of observations at hand (we express thanks to Mr. Takahashi, director of Nagoya Local Meteorological Observatory, for his generous offering the data) and obtained 1.5 and 2.1 for η of Equ. (18). An example is shown in Fig. 7. Comparing these values of η with Fig. 6, it is appropriate to regard that medium diameter falls in 1 to 4*mm*.

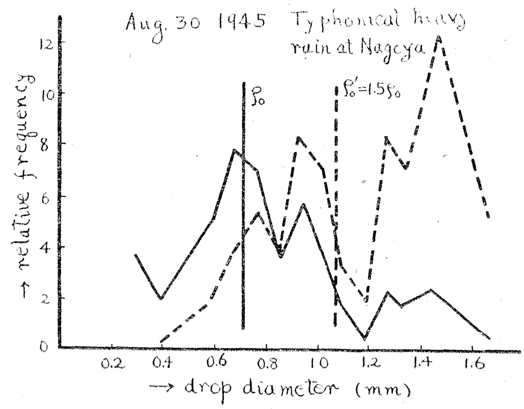


Fig. 7.

In the next place, as for the correlationship between rain drop size and rain fall velocity, theoretically it can be obtained from Stokes's and Noewton's resistance laws but actually it approaches to Newton's law when the drop size is large and to Stokes's law when it is small. In most cases, measurements of Mr. Schmidt of Austraria, or the experimental formula which is a combination of two theories is in use. This relation between drop diameter and rain fall velocity is shown in Fig. 8.

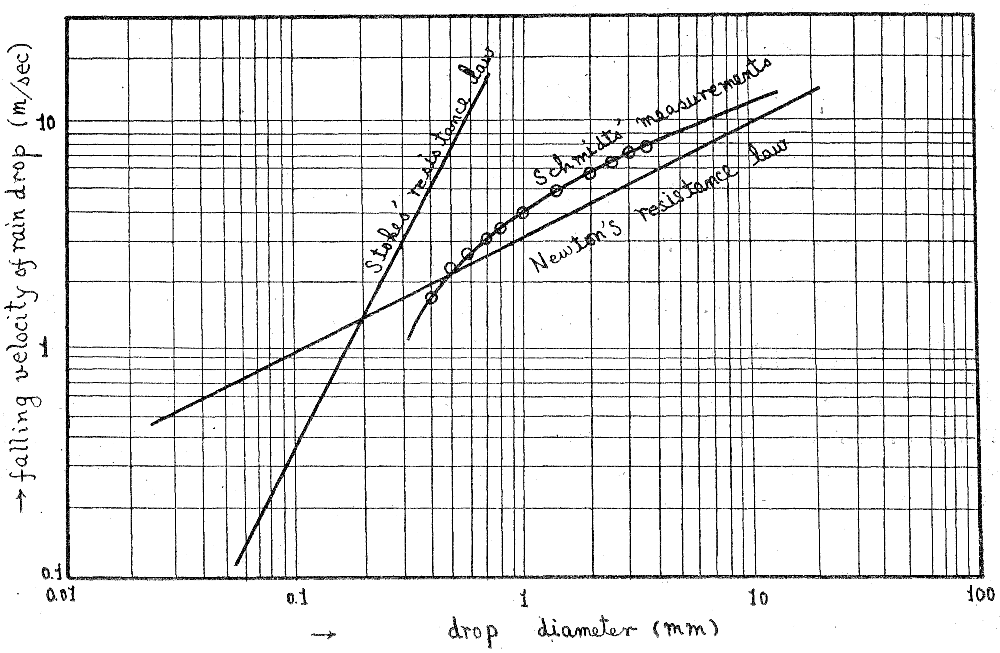


Fig. 8.

IV. Theoretical Value of Attenuation.

As we mentioned in the preceding section, medium diameter effective for the attenuation is 1-4 *mm*, and from Fig. 8. rain fall velocity v *m/sec* is decided, so concentration of rain drops C gr/cm^3 will be found from precipitation h *mm/hour* as follows;

$$C = \frac{0.1h}{60\times 60\times v\times 100} \qquad gr/cm^3 \tag{19}$$

and C is also given by

$$C = N\frac{4}{3}\pi\rho^3.$$

Therefore

$$k = N\frac{\lambda^2}{2\pi} Im\{\sum_{\nu}^{\infty}(-1)^{\nu}(a_\nu - p_\nu)\} = \frac{C\lambda^2}{\frac{4}{3}\pi\rho^3\cdot 2\pi} Im\{\sum_{\nu}^{\infty}(-1)^{\nu}(a_\nu - p_\nu)\}$$

$$= \frac{3C\lambda^2}{8\pi^2\rho^3} Im\{\sum_{\nu}^{\infty}(-1)^{\nu}(a_\nu - p_\nu)\}$$

$$k' = N\frac{\lambda^2}{2\pi}\sum_{\nu=1}^{\infty}\frac{|a_\nu|^2+|p_\nu|^2}{2\nu+1} = \frac{3C\lambda^2}{8\pi^2\rho^3}\sum_{\nu=1}^{\infty}\frac{|a_\nu|^2+|p_\nu|^2}{2\nu+1},$$

so, putting (19) in these,

$$k = \frac{3\lambda^2}{8\pi^2\rho^3}\cdot\frac{h}{36\cdot 10^5 v} Im\{\sum_{\nu=1}^{\infty}(-1)_{\nu}(a_\nu - p_\nu)\} \tag{20}$$

$$k' = \frac{3\lambda^2}{8\pi^2\rho^3}\cdot\frac{h}{36\cdot 10^5 v}\sum_{\nu=1}^{\infty}\frac{|a_\nu|^2+|p_\nu|^2}{2\nu+1}, \tag{21}$$

where k, k' are given in *naper/cm*. Reducing these into *db/mile* which are adopted in measurements,

$$k_0 = 10log_{10}e\times 1.61\times\frac{\lambda^2 h}{12\cdot 8\pi^2 v\rho^3} Im\{-a_1+p_1+a_2-p_2\}$$

$$=\{\gamma_{a1}+\gamma_{p1}+\gamma_{a2}+\gamma_{p2}\}h = \gamma h \tag{22}$$

$$k'_0 = 10log_{10}e\times 1.61\times\frac{\lambda^2 h}{12\cdot 8\pi^2 v\rho^3}\left\{\frac{|a_1|^2}{3}+\frac{|p_1|^2}{3}+\frac{|a_2|^2}{5}+\frac{|p_2|^2}{5}\right\}$$

$$=\{\gamma'_{a1}+\gamma'_{p1}+\gamma'_{a2}+\gamma'_{p2}\}h = \gamma' h \tag{23}$$

Therefore, the attenuation is theoretieally always proportional to the precipitation. γ_{a1}, γ_{p1}, γ_{a2}, γ_{p2}, and γ represent total attenuation involving absorption and scattering due to electric dipole, magnetic dipole, electric quadrupole, magnetic quadrupole and the total effects of these factors, respectively, and γ'_{a1}, γ'_{p1}, γ'_{a2}, γ'_{p2} and γ' represent the attenuation due to only scattering of the corresponding factors. From Equs. (22) and (23) and Table 1 and 2 we can obtain γ,s and γ,'s as in Table 3 and 4 and graphically as shown in Figs. 9. 10. 11. 12, and 13.

Wave length	Drop diameter 2ρ	$1mm$	$1.9mm$	$3.1mm$	$3.7mm$	$4.0mm$	$7.0mm$
	$10log_{10}e\cdot 1.61\lambda^2/12\cdot 8\pi^2 v\rho^3$	5.64	0.595	0.110	0.0600	0.0460	0.00667
	$\times Im\{-a_1\}=\gamma_{a1}$	0.360	0.668	0.183	0.0863	0.0582	0.0073
$0.62cm$	$\times Im\{p_1\}=\gamma_{p1}$	0.294	0.150	0.0912	0.0788	0.0731	0.0129
	$\times Im\{a_2\}=\gamma_{a2}$	0.00373	0.0143	0.0838	0.1019	0.0978	0.0087
	$\times Im\{-p_2\}=\gamma_{p2}$	0.00513	0.0215	0.0263	0.0239	0.0246	0.0254

	$\gamma=\gamma_{a1}+\gamma_{p1}+\gamma_{a2}+\gamma_{p2}$	0.663	0.854	0.384	0.291	0.254	0.0543
1.09*cm*	$10log_{10}e\cdot1.61\lambda^2/128\cdot\pi^2v\rho^3$	17.14	1.84	0.339	0.184	0.142	0.0206
	γ_{a1}	0.0701	0.154	0.287	0.240	0.208	0.0243
	γ_{p1}	0.1021	0.153	0.0677	0.0557	0.0521	0.0399
	γ_{a2}	0.000724	0.00105	0.00412	0.00671	0.0103	0.0352
	$\gamma=\gamma_{a1}+\gamma_{p1}+\gamma_{a2}$	0.173	0.308	0.359	0.302	0.270	0.0994
1.25*cm*	$10loge\cdot1.61\lambda^2/12\cdot8\pi^2v\rho^3$	22.92	2.42	0.446	0.243	0.187	0.0271
	γ_{a1}	0.0493	0.0960	0.224	0.228	0.215	0.0398
	γ_{p1}	0.0717	0.150	0.0658	0.0527	0.0487	0.0318
	γ_{a2}	0.00008	0.00052	0.00198	0.00370	0.00494	0.0320
	$\gamma=\gamma_{a1}+\gamma_{p1}+\gamma_{p2}$	0.121	0.247	0.292	0.284	0.269	0.104
3.2*cm*	$10log_{10}e\cdot1.61\lambda^2/12\cdot8\pi^2v\rho^3$	150.1	15.86	2.92	1.59	1.22	0.178
	γ_{a1}	0.00509	0.00535	0.00762	0.0108	0.0132	0.0432
	γ_{p1}	0.00382	0.0118	0.0526	0.0821	0.0770	0.0200
	γ_{a2}	0.000277	0.000	0.000187	0.000065	0.00004	0.00037
	$\gamma=\gamma_{a1}+\gamma_{p1}+\gamma_{a2}$	0.00919	0.0171	0.0604	0.0930	0.0902	0.0636

Table 3.

Wave length	Drop diame-ter 2ρ	1*mm*	1.9*mm*	3.1*mm*	3.7*mm*	4.0*mm*	7.0*mm*
0.62*cm*	γ'_{a1}	0.145	0.432	0.130	0.060	0.038	0.005
	γ'_{p1}	0.005	0.038	0.061	0.061	0.059	0.011
	γ'_{a2}	0.000	0.002	0.039	0.059	0.060	0.004
	γ'_{p2}	0.000	0.000	0.004	0.008	0.011	0.022
	$\gamma'=\gamma'_{a1}+\gamma'_{p1}+\gamma'_{a2}+\gamma'_{p2}$	0.150	0.472	0.234	0.188	0.168	0.042
1.09*cm*	γ'_{a1}	0.013	0.077	0.190	0.172	0.152	0.017
	γ'_{p1}	0.000	0.007	0.017	0.022	0.024	0.028
	γ'_{a2}	0.000	0.000	0.001	0.002	0.002	0.023
	$\gamma'=\gamma'_{a1}+\gamma'_{p1}+\gamma'_{a2}$	0.013	0.084	0.208	0.196	0.179	0.068
1.25*cm*	γ'_{a1}	0.008	0.045	0.139	0.155	0.151	0.029
	γ'_{p1}	0.000	0.004	0.012	0.015	0.017	0.025
	γ'_{a2}	0.000	0.000	0.000	0.001	0.001	0.019
	$\gamma'=\gamma'_{a1}+\gamma'_{p1}+\gamma'_{a2}$	0.008	0.049	0.151	0.171	0.169	0.073

3.2cm	γ'_{a1}	0.000	0.001	0.003	0.005	0.007	0.026
	γ'_{p1}	0.000	0.000	0.000	0.000	0.000	0.000
	γ'_{a2}	0.000	0.000	0.000	0.000	0.000	0.000
	$\gamma'=\gamma'_{a1}+\gamma'_{p1}+\gamma'_{a2}$	0.000	0.001	0.003	0.005	0.007	0.026

Table 4.

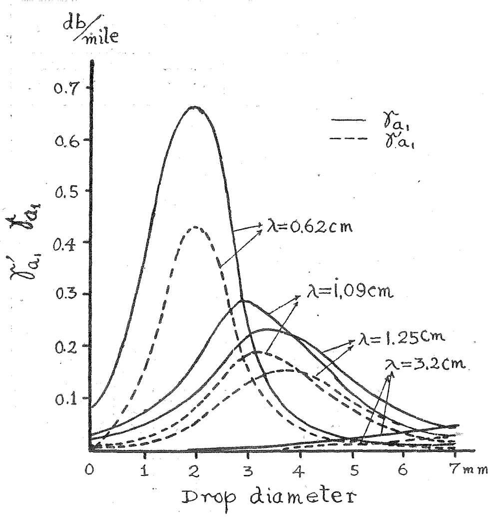


Fig. 9.

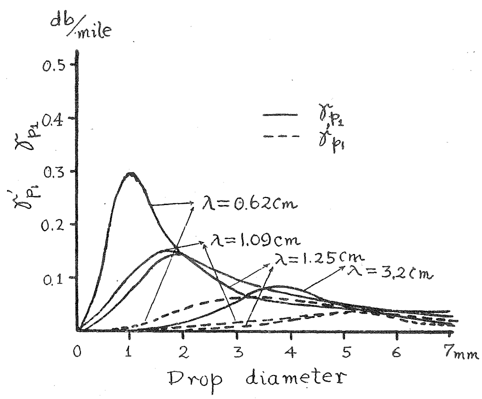


Fig. 10.

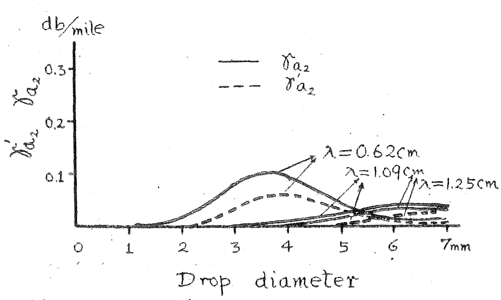


Fig. 11.

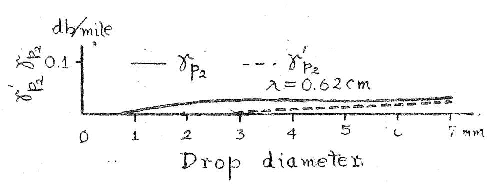


Fig. 12.

According to these, the attenuation by no means increases proportionally to ρ^3 as in the case of Rayleigh scattering, but γ_{a1} shows a maximal attenuation about $2\rho \doteqdot \frac{1}{3}\lambda$, γ_{p1}, about $2\rho \doteqdot \frac{1}{6}\lambda$, and γ_{a2} about $2\rho \doteqdot \frac{2}{3}\lambda$, and amplitude of $\gamma_{a1}>$ amplitude of $\gamma_{p1}>$ amplitude of γ_{a2}. In the case of our wave length and drop size, our approximation to calculate the attenuation up to the a_2 -term or p_2 -term will be almost sufficient.

To compare the theoretical results with the mesurement of Fig. 1 to 4, the lines of Equation (22) $k_0=\gamma h$ obtained from Table 3 are drawn in the corresponding figures as shown in Fig. 14, 15, 16, and 17.

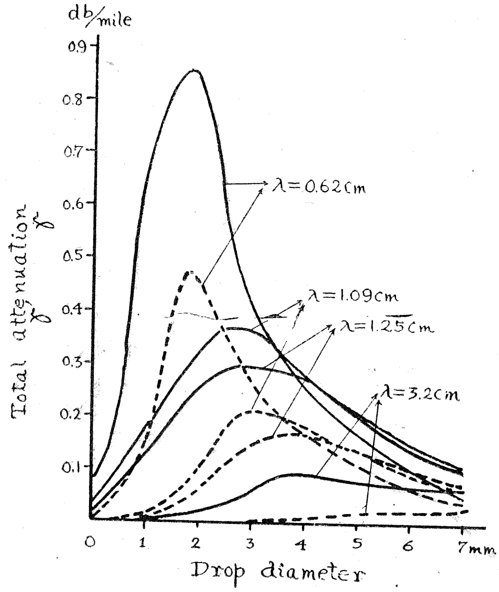


Fig. 13.

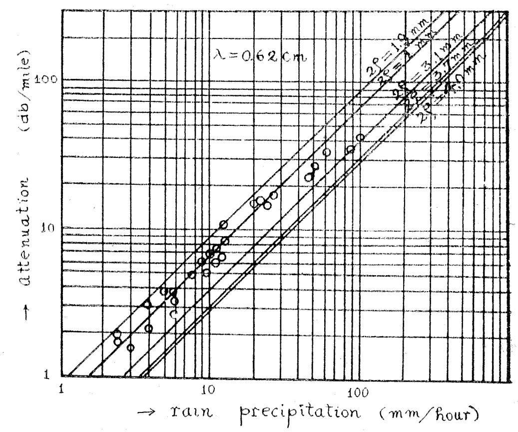


Fig. 14

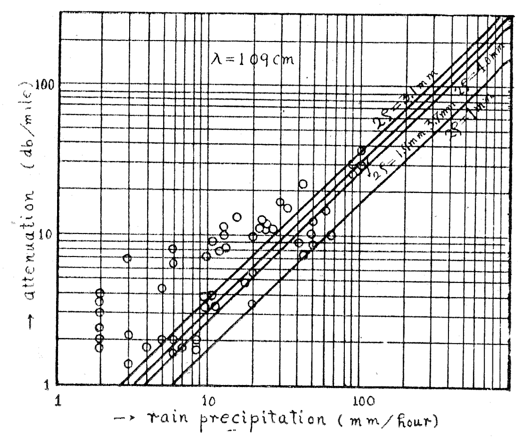


Fig. 15.

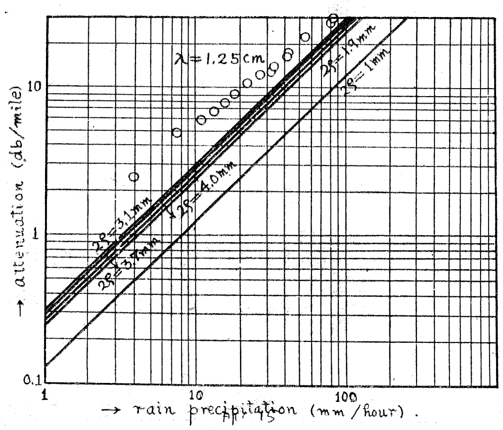


Fig. 16.

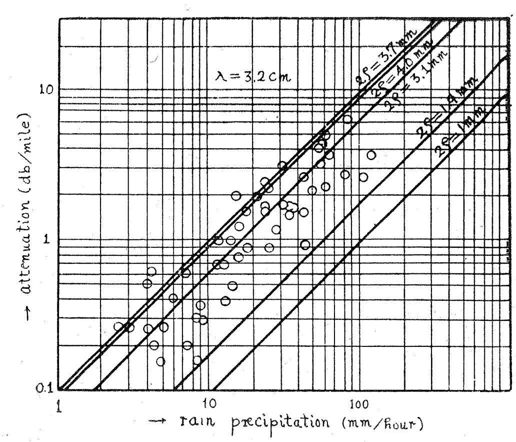


Fig. 17.

V. Comparison of Theory with Experiments and General Consideration.

Comparing the theory and the experiments on Figs. (14) to (17) these are found in good agreement in the order of magnitude. And in the experments the attenuation is proportional to the precipitation, in harmony with the theory.

Especially for the 0.62*cm* and the 3.2*cm* waves, the theoretical attenuation shows very good agreement with the observation. In Fig. 16 for the 1.25*cm* wave, observed volues are a little greater than the theoretical attenuation.

For the 1.09*cm* wave, we see in Fig 15 better agreement than the 1.25*cm* wave, but a slight discrepancy exists in low precipitation.

We have calculated hitherto only electric dipole, magnetic dipole and electric quadrupole terms exactly and for 0.62*cm* wave magnetic quadrupole also, so we will discuss the points which is to be considered for further exactness.

Firstly as for the calculation of the attenuation constant according to the assumption employed by Mie;

1. Assumption of sphere shaped dielectrics appears to be proper for the case of rain. But the assumption that k is N times that of one particle, is not always right for our case, but comes into question at two points as follows:
2. Rain drops receive the secondary waves reflected from other drops in addition to the incident plane wave, so that there exist so called multiple diffraction.
3. Putting the scattered electric and magnetic fields from *i-th* individual rain drop to $\mathbb{E}_i$ and $\mathbb{H}_i$, the total energy lost by scattering is

$$L=\int_0^1 dt\int\int(\sum_i \mathbb{E}_i)\times(\sum_i \mathbb{H}_i)dw \tag{24}$$

and this is not equal to the sum of individual scattered energy

$$L'=\int_0^1 dt\int\int\sum_i(\mathbb{E}_i\times\mathbb{H}_i)dw \tag{25}$$

For these points there is the researches by Ros Gans and the others, and further discussions are necessary for our case, but, for the first approximation assumption of N times will be still proper.

Secondly for the refractive index of the water;

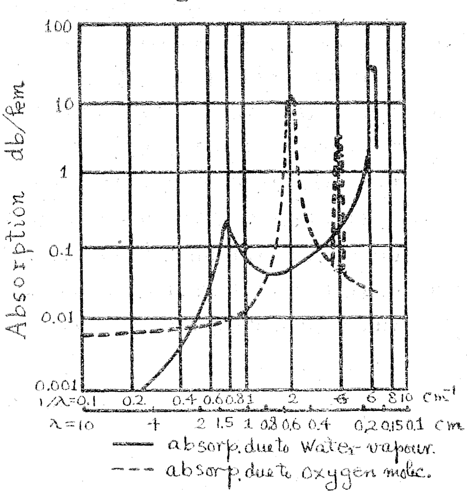
4. Although Debye's theory have been verified by mesurements, both the theory and the measurements deal with the water as vapour or dilute solution in an other solvent. In the actual water, however, there is some association, and it seems not proper to employ the vapour value of refractive index. In this place, however, it is difficult to find the value which include the existence of association. The values $|\varepsilon|$ and $\tan\delta$ used in our calculation, listed in Table 1. are derived from the measurements by A. Esaw and G. Bäz at temperater about 20°C. But it is well known that the dielectric constant and $\tan\delta$ depend on the temperature. All our four measure-

ments of the attenuation of microwaves by rain drops are reported in April, and so the air temperature at the time of the measurements will be not far from 20°C. But the rain drop temperature at the time of individual measurements may perhaps distribute considerably. So we calculate the values of $|\varepsilon|$ and $tan\delta$ at temperature 25°C, 20°C, 10°C and 0°C after the Debye's formulae, Collies, Hasted and Ritson's[8] measurements, and Haggis, Hasted and Buchanan's[9] for the conference as follows,

	$\|\varepsilon\|_{0°}$	$tan\delta_{0°}$	$\|\varepsilon\|_{10°}$	$tan\delta_{10°}$	$\|\varepsilon'\|_{20°}$	$tan\delta_{20°}$	$\|\varepsilon\|_{25°}$	$tan\delta_{25°}$
0.62*cm*	18.3	2.58	21.5	2.30	25.0	1.96	28.5	1.83
1.09	29.5	2.14	34.0	1.84	40.0	1.48	45.0	1.27
1.25	33.5	1.97	38.5	1.63	44.6	1.36	49.0	1.11
3.2	63.3	0.92	68.5	0.71	71.0	0.55	70.0	0.46

5. For our cace, since $\alpha > 1$, $\beta > 1$ higher mode terms a_3, a_4,$\cdots$ p_3, $p_4\cdots$ may become essential. Thirdly concerning to the treatment of drop size and precipitation.
6. The rain drop size employed here is not that of the case when the measurement was carried out, but estimated from the other data, and therefore it may be possible that the estimation is not appropriate.
7. As it is mentioned in the papers of measurements, the precipitation is not uniform throughout the path in which the measurements are carried out, and the measured precipitation does not represent what actually contributed to the attenuation, without sufficient number of measuring point.
8. Since it is difficult to measure instantaneous the precipitation of the rain fall which varies rapidly with time, the precipitation at the instant of mesurement doesn't coincide with the mean precipitation before and after the time. In addition to these aspects.
9. In the centimetre and millimetre wave region, the precise measurement of attennation is considerably difficult, and it seems unable to avoid the inclusion of more or less systematic error.

Fig. 18.



On the discrepancy of the theory with the experiments for the 1.25*cm* waves, we must notice the facts that the observed attenuation for the 1.25*cm* wave falls on almost the same line of that of the 0.62*cm* wave. and that we have ignored entirely the attenuation due to the water vapour contents involved in the wave path, because we have no knowledge of the humidity variation in the rain fall. But the water vapour molecules have selective absorption at about $\lambda = 1.35\ cm$ according to Van Vleck[10] and others. Fig. 18 are reproduced from Van Vleck's paper. The attenuation due to the water vapour molecules with the full line which are contained 1% mass ratio ($7.5gr/m^3$) in the air and that due to the oxygen molecules with the dotted line are shown. At temperature about 20°C and the relative humidity 100%, about $18gr/m^3$ of the water vapour molecules is contained in the air, and so the effect on the attenutattion due to these is less than $0.5db/mile$ for $1.25cm$ wave, and for $0.62cm$ that due to the water vapour molecules and oxygen molecules is less than $0.8db/mile$ from Fig. 18. Therefore we can conclude as the result, that the selective absorptions due to the water vapour molecules and the oxygen molecules have no appreciable effects in our case.

VI. Conclusion.

As the phenomena of absorption and scattering of centimeter and millimeter waves by rain drops are very interesting in the present state that the application of microwaves to the meteorology is of paramount importance, and as the rain drop size amounts to the same order with the wave length, we have calculated exactly the electric dipole, the magnetic dipole, clectric quadrupole, and the magnetic quadrupole terms of absorption and scattering, going back to the Mie's paper; and deduced the concentration of rain drop, assuming the drop diameter as 1 to $4mm$, from various data; and compared the theoretical attenuation with the experiments in U. S. A. As the result, the experiments agree well with the theory.

But on the slight discrepancy for $1.25cm$ and $1.09cm$ waves, we have discussed various conceivable origins of errors.

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