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Classifications of Lie Groups Realized on Coxeter Orbifolds

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Abstract : A procedure is explaind for obtaining the gauge groups and the representations of massless untwisted matter fields realized on Coxeter (Z_N) orbifolds. All the results are listed in tables, except for untwisted matter representations of Z_{12}

1 Introduction

Coxeter (Z_N) orbifold compactification [1] is one of the most interesting approach to obtain a realistic four dimensional theory from the $E_8 \times E_8'$ heterotic string theory [2]. The Z_N orbifold is obtained in such a way to divide a six-dimensional torus by a discrete rotation Z_N. The order of discrete rotation N should be equal to 3,4,6,7,8 or 12 in order to preserve N=1 supersymmetry [3].

The Z_3 orbifold models are well known to be classified into four models [1]. Remaining orbifold models have been also classified systematically, and recipes to obtain all the massless physical states have been discussed, but most of the matter contents have not been listed so as not to occupy too many pages [4, 5, 6].

In this paper, we list all the gauge groups and the representations of massless untwisted matter fields realized on Z_N orbifolds without Wilson-line mechanism, except for matter fields of Z_{12}.

The outline of this paper is as follows. In section 2, constraints for shift vectors and massless physical states of Z_N orbifold models are discussed. The procedure is explained for acquiring unbroken gauge groups and untwisted matter representations in section 3. The last section is devoted in summary.

2 Constraints

The $E_8 \times E_8'$ heterotic string consists of 26 dimensional left-moving bosonic string and 10 dimensional right-moving superstring. The left-movers are composed of space-time coordinates X_L^i ($i = 1 \sim 8$ in the light-cone gauge) and internal coordinates X^I ($I = 1 \sim 16$). The right-movers are composed of space-time coordinates X_R^i and fermionic coordinates ψ_R^i. It is possible to describe the fermionic coordinates ψ_R^i by NSR coordinate ϕ^t ($t = 1 \sim 4$) in bosonic forms. We use the bosonic forms, hereafter. The bosonic coordinates X^I and ϕ^t are compactified on $E_8 \times E_8'$ root lattice $\Gamma_{E_8 \times E_8'}$ and $SO(8)$ weight lattice $\Gamma_{SO(8)}$, respectively. The 6-dimensional coordinates $X^j_{\rm L}$ and $X^j_{\rm R}\,(j=3\sim 8)$ should also be compactified to construct 4-dim. theory.

The Z_N orbifold preserving N=1 supersymmetry are obtained with the division of a 6-dimesional torus by a discrete rotation Z_N, which can always be diagonalized as

$$\theta = \text{diag.}[\exp(2\pi i\eta^a)] \quad a = 1\sim 3,$$

under complex basis. The values of exponents η^a of each orbifold are written in the second column of Table 1. When $X^j_{\rm L}$ and $X^j_{\rm R}\,(j=3\sim 8)$ are divided by θ, $\psi^j_{\rm R}$ should be divided simultaneously by θ in order to preserve worldsheet supersymmetry. In the bosonic form $8_{\rm v}$ or $8_{\rm s}$ weight vector $p^t\,(t=1\sim 4)$ of $\Gamma_{SO(8)}$ should be divided by a shift v^t such as

$$p^t \sim p^t + v^t,$$

where

$$(v^1,\, v^2,\, v^3,\, v^4) = (\eta^1,\, \eta^2,\, \eta^3,\, 0).$$

The $E_8\times E_8'$ root $P^I\,(I=1\sim 16)$ of $\Gamma_{E_8\times E_8'}$ should be also divided simultaneouly by an automorphism Θ^{IK}:

$$P^I \sim \Theta^{IK} P^K,$$

or a shift V^I:

$$P^I \sim P^I + V^I.$$

It is pointed out that embedding in terms of an automorphism is always realized in terms of a shift [7], but the converse is not true. Therefore, we will investigate all the independent shifts.

Since the shift of $\Gamma_{E_8\times E_8'}$ is equivalent to a rotation of $16+16$ RNS fermionic coordinates, it must respect an algebraic requirement of $SO(16)\times SO(16)$:

$$N\sum_{J=1}^{8} V^J = N\sum_{J'=9}^{16} V^{J'} = 0 \pmod 2. \tag{1}$$

Modular invariance further restricts the shift as

$$N\sum_{t=1}^{4}(v^t)^2 = N\sum_{J=1}^{8}(V^J)^2 = N\sum_{J'=9}^{16}(V^{J'})^2 = 0 \pmod 2. \tag{2}$$

There exist two types of closed strings on the orbifold, i.e., untwisted strings and twisted strings. Mass formulas for untwisted strings are

$$\begin{aligned} \frac{1}{8}(m^{(0)}_{\rm R})^2 &= \frac{1}{2}\sum_{j=3}^{8}(p^j_{\rm R})^2 + \frac{1}{2}\sum_{t=1}^{4}(p^t)^2 + N^{(0)}_{\rm R} - \frac{1}{2}, \\ \frac{1}{8}(m^{(0)}_{\rm L})^2 &= \frac{1}{2}\sum_{j=3}^{8}(p^j_{\rm L})^2 + \frac{1}{2}\sum_{I=1}^{16}(P^I)^2 + N^{(0)}_{\rm L} - 1, \end{aligned} \tag{3}$$

where $N^{(0)}_{\rm R}$ and $N^{(0)}_{\rm L}$ represent number operators of the untwisted sector. Modular invariance requirs the Mass-level matching condition $m^2_{\rm R} = m^2_{\rm L}$ [8].

A further constraint is derived from a modular invariant partition function, namely one-loop vaucum amplitude. Physical states of the untwisted string are selected by the generalized GSO projection operator [9, 10]:

$$P_0 = \frac{1}{N}\sum_{l=0}^{N-1} \Delta_0^l, \tag{4}$$

where

$$\Delta = \exp 2\pi i[\sum_{I=1}^{16} P^I V^I - \sum_{t=1}^{4} p^t v^t]. \tag{5}$$

Massless physical states in untwisted sectors can be found from these constraints as explained in the next section.

3 Untwisted sector

The massless untwisted sectors are composed of gauge supermultiplets, untwisted matter supermultiplets, 4-dim. supergravity multiplet and gauge-singlet matter supermultiplets. Physical states conditions are derived from eqs.(4)(5) for the gauge supermultiplets:

$$\sum_{I=1}^{16} P^I V^I \in \mathbf{Z} \quad \text{and} \sum_{t=1}^{4} p^t v^t \in \mathbf{Z}, \tag{6}$$

and for the untwisted matter multiplets:

$$\sum_{I=1}^{16} P^I V^I - \sum_{t=1}^{4} p^t v^t \in \mathbf{Z} \quad \text{and} \sum_{t=1}^{4} p^t v^t \notin \mathbf{Z}. \tag{7}$$

If $V^J\,(V^{J'})$ respecting eq.(1) is given, an unbroken gauge groups of an observed sector (hidden sector) is found from $P^J(P^{J'})$ of $E_8(E_8')$ root satisfying eq.(6) when $p^t = (0,0,0,\pm1)$. It is noted that the massless condition of eq.(3) is satisfied with $\sum(P^J)^2 + \sum(P^{J'})^2 = 2$ and $\sum(p_{\rm L}^j)^2 = N_{\rm L}^{(0)} = 0$ or $\sum(P^J)^2 + \sum(P^{J'})^2 = \sum(p_{\rm L}^j)^2 = 0$ and $N_{\rm L}^{(0)} = 1$. Table 2 contains all the gauge groups of the observed sector or the hidden sector in Z_N orbifold models. [1]

The representations of untwisted matters are found from the simple roots of unbroken gauge groups and the E_8 roots P^J ($E_{8'}$ roots $P^{J'}$) satisfying eq.(7) when $p^t = (\pm1,0,0,0)$, $p^t = (0,\pm1,0,0)$, or $p^t = (0,0,\pm1,0)$. All the representations are listed in Table 3, 4, 5, 6 and 7, except for those of Z_{12} orbifold. When a gauge group and its untwisted matters are also realized in terms of automorphism, it is denoted by superscript A of the corresponding shift.

The modular invariance eq.(2) restricts the combinations of V^J and $V^{J'}$, then it also restricts the combinations of gauge groups originating from E_8 and E_8'.

[1] There is an alternative way to know the unbroken gauge groups. It is more intuitive and diagramatical, as reviewed in Ref.[6].

4 Summery

In this paper we have discussed the procedure for obtaining the unbroken gauge groups and the representations of massless matter superfields in untwisted sectors. The results obtained by this procedure are written in the Tables, where we have omitted $U(1)$ charges of the other matter representations.

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Table 1. 6-dim. Lattices in Z_N Orbifolds

Numbers (numbers of parentheses) in the fourth and fifth columns denote realizations in terms of shift (automorphism) of $E_8 \times E_8'$ lattice. Unbroken Gauge group ($E_8 \times E_8$) are denoted by $\star$. The superscript [n] in the third column is equal to the order of an outer automorphism.

Point Group	Exponent η	Lie Lattice	No. of Gauge Gruops	No. of Indep. Models
Z_3	(1,1,-2)/3	$(SU_3)^3$	4 (4)+$\star$	4 (4)+$\star$
Z_4	(1,1,-2)/4	$(SU_4)^2$ $SO_5 \times SU_4 \times SU_2$ $(SO_5)^2 \times (SU_2)^2$	12 (12)	12 (12)
Z_6-I	(1,1,-2)/6	$(G_2)^2 \times SU_3$ $(SU_3^{[2]})^2 \times SU_3$	48 (26)	58 (26)
Z_6-II	(1,2,-3)/6	$SU_6 \times SU_2$ $SO_8 \times SU_3$ $SO_7 \times SU_3 \times SU_2$ $G_2 \times SU_3 \times (SU_2)^2$ $SU_3^{[2]} \times SU_3 \times (SU_2)^2$ $SU_4^{[2]} \times SU_3 \times SU_2$	54 (28)	61 (28)
Z_7	(1,2,-3)/7	SU_7	39 (2)+$\star$	39 (2)+$\star$
Z_8-I	(1,2,-3)/8	$SO_9 \times SO_5$ $SO_8^{[2]} \times SO_5$	119 (25)	246 (25)
Z_8-II	(1,3,-4)/8	$SO_8^{[2]} \times (SU_2)^2$ $SO_{10} \times SU_2$ $SO_9 \times (SU_2)^2$	120 (24)	248 (24)
Z_{12}-I	(1,4,-5)/12	E_6 $F_4 \times SU_3$ $SO_8^{[3]} \times SU_3$	581 (92)	3026 (96)
Z_{12}-II	(1,5,-6)/12	$SO_4 \times F_4$ $SO_8^{[3]} \times (SU_2)^2$	603 (110)	3013 (112)

Table 2-1. Gauge Groups in Z_N orbifold models

Numbers in the last six columns denote independent shifts of E_8 lattice. Among them, ones equivalent to automorphism are denoted by numbers of parentheses.

No.	Gauge Groups	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}
0	E_8	⋆	⋆	⋆	⋆	⋆	⋆
1	$E_7 \times SU_2$		1 (1)	1 (1)		1 (1)	1 (1)
2	$E_7 \times U_1$	1 (1)	1 (1)	2 (1)	3	3 (1)	5 (2)
3	$E_6 \times SU_3$	1 (1)		1 (1)			1 (1)
4	$E_6 \times SU_2 \times U_1$		1 (1)	1	3	3 (1)	4 (1)
5	$E_6 \times U_1^2$			1 (1)	1	2	7 (1)
6	SO_{16}		1 (1)	1 (1)		1 (1)	1 (1)
7	$SO_{14} \times U_1$	1 (1)	1 (1)	2 (1)	3	3 (1)	5 (2)
8	$SO_{12} \times SU_2 \times U_1$		1 (1)	2 (1)		3 (1)	5 (2)
9	$SO_{12} \times U_1^2$			1	3	3 (1)	10 (1)
10	$SO_{10} \times SU_4$		1 (1)			1 (1)	1 (1)
11	$SO_{10} \times SU_3 \times U_1$			1	3	2	3
12	$SO_{10} \times SU_2^2 \times U_1$			1 (1)		1 (1)	2 (1)
13	$SO_{10} \times SU_2 \times U_1^2$			1 (1)	3	4	17 (3)
14	$SO_{10} \times U_1^3$					1 (1)	9
15	$SO_8 \times SU_4 \times U_1$			1 (1)		1 (1)	2 (1)
16	$SO_8 \times SU_3 \times U_1^2$				1 (1)	1 (1)	5
17	$SO_8 \times SU_2^2 \times U_1^2$					1 (1)	4 (1)
18	$SO_8 \times SU_2 \times U_1^3$						5 (1)
19	$SO_8 \times U_1^4$						1 (1)
20	SU_9	1 (1)		1 (1)			1 (1)
21	$SU_8 \times SU_2$		1 (1)			1 (1)	1 (1)
22	$SU_8 \times U_1$		1 (1)	2 (1)	3	4 (1)	6 (2)
23	$SU_7 \times SU_2 \times U_1$			1	3	2	3
24	$SU_7 \times U_1^2$			2 (1)	3	5	19 (1)
25	$SU_6 \times SU_3 \times SU_2$			1 (1)			1 (1)

continued...

Table 2-2. Gauge Groups in Z_N orbifold models

No.	Gauge Groups	Z_3	Z_4	Z_6	Z_7	Z_8	Z_{12}
26	$SU_6 \times SU_3 \times U_1$			1 (1)			4 (2)
27	$SU_6 \times SU_2^2 \times U_1$			1 (1)		2	3 (1)
28	$SU_6 \times SU_2 \times U_1^2$				3	5 (1)	18 (1)
29	$SU_6 \times U_1^3$					1 (1)	16 (1)
30	$SU_5 \times SU_4 \times U_1$			1 (1)	3	2	3 (1)
31	$SU_5 \times SU_3 \times SU_2 \times U_1$				3	2	2
32	$SU_5 \times SU_3 \times U_1^2$					3 (1)	12
33	$SU_5 \times SU_2^2 \times U_1^2$					1 (1)	13
34	$SU_5 \times SU_2 \times U_1^3$						19
35	$SU_5 \times U_1^4$						3
36	$SU_4^2 \times SU_2 \times U_1$					1 (1)	2 (1)
37	$SU_4^2 \times U_1^2$					1 (1)	8 (1)
38	$SU_4 \times SU_3 \times SU_2^2 \times U_1$					1 (1)	1
39	$SU_4 \times SU_3 \times SU_2 \times U_1^2$						7
40	$SU_4 \times SU_3 \times U_1^3$						13
41	$SU_4 \times SU_2^3 \times U_1^2$						5 (2)
42	$SU_4 \times SU_2^2 \times U_1^3$						9 (4)
43	$SU_4 \times SU_2 \times U_1^4$						1 (1)
44	$SU_3^3 \times SU_2 \times U_1$						1 (1)
45	$SU_3^3 \times U_1^2$						1 (1)
46	$SU_3^2 \times SU_2^2 \times U_1^2$						3 (1)
47	$SU_3^2 \times SU_2 \times U_1^3$						4 (2)
48	$SU_3 \times SU_2^4 \times U_1^2$						1 (1)
49	$SU_3 \times SU_2^3 \times U_1^3$						1 (1)
Total # of Gauge Groups		4 (4)	9 (9)	21 (17)	14 (1)	30 (22)	49 (37)
Total # of Shift (Auto.)		4 (4)	9 (9)	26 (17)	38 (1)	62 (22)	269 (49)

Table 3. Gauge groups and untwisted matters in Z_3 orbifold models

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism. The values of $\sum P^J V^J$ include mod 1. Untwisted matters satisfying $\sum P^J V^J = 2/3$ mod 1 are antichiral and conjugate (*i.e.*, antipar ticles) of those satisfying $\sum P^J V^J = 1/3$ mod 1.

No.	Shift	Gauge Group	Untwisted Matters
	$3V^J$	$\sum P^J V^J = 0/3$	1/3
0	$(00000000)^A$	E_8	
1	$(11000000)^A$	$E_7 \times U_1$	3(56)+3(1)
2	$(21100000)^A$	$E_6 \times SU_3$	$3(27,3)$
3	$(20000000)^A$	$SO_{14} \times U_1$	$3(64_c) + 3(14_v)$
4	$(21111000)^A$	SU_9	3(84)

Table 4. Gauge groups and untwisted matters in Z_4 orbifold models

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism. The values of $\sum P^J V^J$ include mod 1. Untwisted matters satisfying $\sum P^J V^J = 3/4$ mod 1 are antichiral and conjugate (*i.e.*, antiparticles) of those satisfying $\sum P^J V^J = 1/4$ mod 1. Representations with $\sum P^I V^I = 1/4$ must be duplicated.

No.	Shift	Gauge Group	Untwisted Matters	
	$4V^J$	$\sum P^J V^J = 0/4$	1/4	2/4
0	$(00000000)^A$	E_8		
1	$(22000000)^A$	$E_7 \times SU_2$		(56,2)
2	$(11000000)^A$	$E_7 \times U_1$	(56)	2(1)
3	$(21100000)^A$	$E_6 \times SU_2 \times U_1$	$(\overline{27},2) + (1,2)$	$(\overline{27},1) + (27,1)$
4	$(40000000)^A$	SO_{16}		(128_s)
5	$(20000000)^A$	$SO_{14} \times U_1$	(64_s)	$2(14_v)$
6	$(31000000)^A$	$SO_{12} \times SU_2 \times U_1$	$(32_c,1) + (12_v,2)$	$(32_c,1) + 2(1,1)$
7	$(22200000)^A$	$SO_{10} \times SU_4$	$(16_c,4)$	$(10_v,6)$
8	$(31111100)^A$	$SU_8 \times SU_2$	(28,2)	(70,1)
9	$(1111111\text{-}1)^A$	$SU_8 \times U_1$	$(\overline{56}) + (8)$	$(28) + (\overline{28})$

Table 5. Gauge groups and untwisted matters in Z_6 orbifold models

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism. The values of $\sum P^J V^J$ include mod 1. Untwisted matters satisfying $\sum P^J V^J = 4/6$ and $5/6$ mod 1 are antichiral and conjugate (*i.e.*, antiparticles) of those satisfying $\sum P^J V^J = 2/6$ and $1/6$ mod 1, respectively. In Z_6-I, representations in $\sum P^J V^J = 1/6$ (which must be duplicated) and $4/6$ are chiral (or antichiral), and those in $\sum P^J V^J = 3/6$ are projected out. In Z_6-II, representations in $\sum P^J V^J$=1/6, 2/6 and 3/6 are chiral (or antichiral).

No.	Shift	Gauge Group	Untwisted Matters		
	$6V^J$	$\sum P^J V^J = 0/6$	1/6	2/6	3/6
0	$(00000000)^A$	E_8			
1	$(33000000)^A$	$E_7 \times SU_2$			(56,2)
2	(11000000)	$E_7 \times U_1$	(56)	(1)	
3	$(22000000)^A$	$E_7 \times U_1$		(56)+(1)	
4	$(42200000)^A$	$E_6 \times SU_3$		$(\overline{27}, \overline{3})$	
5	(21100000)	$E_6 \times SU_2 \times U_1$	$(\overline{27}, 2)$	(27,1)	2(1,2)
6	$(32100000)^A$	$E_6 \times U_1^2$	$(\overline{27})$+(1)+(1)	$(\overline{27})$+(1)	(27)+$(\overline{27})$
7	$(60000000)^A$	SO_{16}			(128_s)
8	(20000000)	$SO_{14} \times U_1$	(64_c)	(14_v)	
9	$(40000000)^A$	$SO_{14} \times U_1$		$(64_c)+(14_v)$	
10	(42000000)	$SO_{12} \times SU_2 \times U_1$	$(32_s, 1)$	$(12_v, 2)$+(1,1)	$(32_c, 2)$
11	$(51000000)^A$	$SO_{12} \times SU_2 \times U_1$	$(12_v, 2)$	$(32_s, 1)$+(1,1)	$(32_c, 2)$
12	(31000000)	$SO_{12} \times U_1^2$	$(32_s)+(12_v)$	(32_c)+2(1)	$2(12_v)$
13	(22200000)	$SO_{10} \times SU_3 \times U_1$	$(16_s, \overline{3})$	$(10_v, 3)$+(1,3)	$(16_s, 1)+(16_c, 1)$
14	$(33200000)^A$	$SO_{10} \times SU_2^2 \times U_1$	$(16_s, 1, 2)$ +(1,2,2)	$(16_s, 2, 1)$ $+(10_v, 1, 1)$	$(10_v, 2, 2)$
15	$(41100000)^A$	$SO_{10} \times SU_2 \times U_1^2$	$(16_c, 1)+(10_v, 2)$ +(1,2)	$(16_s, 2)+(10_v, 1)$ +(1,1)	$(16_c, 2)+(16_s, 1)$ +2(1,2)
16	$(51110000)^A$	$SO_8 \times SU_4 \times U_1$	$(8_c, 4)+(8_v, 1)$	$(8_s, 4)$+(1,6)	$(8_v, 6)$
17	$(51111111)^A$	SU_9		(84)	
18	(1111111-1)	$SU_8 \times U_1$	$(\overline{56})$	(28)	(8)+$(\overline{8})$
19	$(5111111\text{-}1)^A$	$SU_8 \times U_1$	$(\overline{28})$+(1)	$(\overline{28})$	(70)
20	(7111111-1)/2	$SU_7 \times SU_2 \times U_1$	(21,2)	$(\overline{35}, 1)+(\overline{7}, 1)$	(7,2)+$(\overline{7}, 2)$
21	(31111111)	$SU_7 \times U_1^2$	(35)+$(\overline{7})$+(1)	(21)+2$(\overline{7})$	(21)+$(\overline{21})$
22	$(91111111)/2^A$	$SU_7 \times U_1^2$	(21)+(7)+$(\overline{7})$	(35)+(7)+(1)	(21)+$(\overline{21})$
23	$(51111100)^A$	$SU_6 \times SU_3 \times SU_2$	$(\overline{6}, \overline{3}, 2)$	$(\overline{15}, 3, 1)$	(20,1,2)
24	$(93311111)/2^A$	$SU_6 \times SU_3 \times U_1$	(20,1)+$(\overline{6}, \overline{3})$	$(15, \overline{3})$+(1,1)	(6,3)+$(\overline{6}, \overline{3})$
25	$(3311111\text{-}1)^A$	$SU_6 \times SU_2^2 \times U_1$	$(\overline{15}, 1, 2)$ +(6,2,1)	(15,1,1) +$(\overline{6}, 2, 2)$	(20,2,1) +2(1,1,2)
26	$(22222000)^A$	$SU_5 \times SU_4 \times U_1$	$(\overline{10}, 4)+(1, \overline{4})$	$(\overline{10}, 1)$+(5,6)	(5,4)+$(\overline{5}, \overline{4})$

Table 6-1. Gauge groups and untwisted matters in Z_7 orbifold models

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism. The values of $\sum P^J V^J$ include mod 1. Untwisted matters satisfying $\sum P^J V^J = 3/7$, $5/7$ and $6/7$ mod 1 are antichiral and conjugate (*i.e.*, antiparticles) of those satisfying $\sum P^J V^J = 4/7$, $2/7$ and $1/7$ mod 1, respectively.

No.	Shift $7V^J$	Gauge Group $\sum P^J V^J = 0/7$	Untwisted Matters 1/7	2/7	4/7
0	$(00000000)^A$	E_8			
1	(11000000)	$E_7 \times U_1$	(56)	(1)	
2	(11111111)	$E_7 \times U_1$		(56)	(1)
3	(33000000)	$E_7 \times U_1$	(1)		(56)
4	(11111100)	$E_6 \times SU_2 \times U_1$	(27, 2)	$(\overline{27}, 1)$	(1,2)
5	(22222200)	$E_6 \times SU_2 \times U_1$	(1,2)	(27, 2)	$(\overline{27}, 1)$
6	(22222211)	$E_6 \times SU_2 \times U_1$	$(\overline{27}, 1)$	(1,2)	(27,2)
7	(22111111)	$E_6 \times U_1^2$	(27)+(1)	(27)+(1)	(27)+(1)
8	(11110000)	$SO_{14} \times U_1$	(64_s)	(14_v)	
9	(22220000)	$SO_{14} \times U_1$		(64_s)	(14_v)
10	(33330000)	$SO_{14} \times U_1$	(14_v)		(64_s)
11	(21111110)	$SO_{12} \times U_1^2$	(32_c)+(12_v)	(32_s)+(1)	(12_v)+(1)
12	(22221111)	$SO_{12} \times U_1^2$	(32_s)+(1)	(12_v)+(1)	(32_c)+(12_v)
13	(33220000)	$SO_{12} \times U_1^2$	(12_v)+(1)	(32_c)+(12_v)	(32_s)+(1)
14	(22111100)	$SO_{10} \times SU_3 \times U_1$	$(16_s, 3)$	$(10_v, \overline{3})$	$(16_s, 1)$+(1,3)
15	(33222200)	$SO_{10} \times SU_3 \times U_1$	$(16_s, 1)$+(1,3)	$(16_s, 3)$	$(10_v, \overline{3})$
16	(33331100)	$SO_{10} \times SU_3 \times U_1$	$(10_v, \overline{3})$	$(16_s, 1)$+(1,3)	$(16_s, 3)$
17	(22221100)	$SO_{10} \times SU_2 \times U_1^2$	$(16_s, 1)$+$(10_v, 2)$	$(16_c, 2)$+(1,2) +(1,1)	$(16_c, 1)$+$(10_v, 1)$ +(1,2)
18	(22222110)	$SO_{10} \times SU_2 \times U_1^2$	$(16_c, 2)$+(1,2) +(1,1)	$(16_c, 1)$+$(10_v, 1)$ +(1,2)	$(16_s, 1)$+$(10_v, 2)$
19	(32222210)	$SO_{10} \times SU_2 \times U_1^2$	$(16_c, 1)$+$(10_v, 1)$ +(1,2)	$(16_s, 1)$+$(10_v, 2)$	$(16_c, 2)$+(1,2) +(1,1)

continued...

Table 6-2. Gauge groups and untwisted matters in Z_7 orbifold models

No.	Shift $7V^J$	Gauge Group $\sum P^J V^J = 0/7$	Untwisted Matters 1/7	2/7	4/7
20	$(3222211\text{-}1)^A$	$SO_8 \times SU_3 \times U_1^2$	$(8_s,3)+(8_v,1)+(1,\overline{3})$	$(8_c,3)+(8_s,1)+(1,\overline{3})$	$(8_v,3)+(8_c,1)+(1,\overline{3})$
21	(1111111-1)	$SU_8 \times U_1$	(56)	$(\overline{28})$	$(\overline{8})$
22	(2222222-2)	$SU_8 \times U_1$	$(\overline{8})$	(56)	$(\overline{28})$
23	(4222222-2)	$SU_8 \times U_1$	$(\overline{28})$	$(\overline{8})$	(56)
24	(2211111-1)	$SU_7 \times SU_2 \times U_1$	$(21,2)$	$(\overline{35},1)$	$(7,2)+(7,1)$
25	(22222220)	$SU_7 \times SU_2 \times U_1$	$(\overline{35},1)$	$(7,2)+(7,1)$	$(21,2)$
26	(3322222-2)	$SU_7 \times SU_2 \times U_1$	$(7,2)+(7,1)$	$(21,2)$	$(\overline{35},1)$
27	(22211110)	$SU_7 \times U_1^2$	$(35)+(\overline{7})$	$(21)+(\overline{7})+(1)$	$(21)+(7)$
28	(3221111-1)	$SU_7 \times U_1^2$	$(21)+(7)$	$(35)+(\overline{7})$	$(21)+(\overline{7})+(1)$
29	(33321000)	$SU_7 \times U_1^2$	$(21)+(\overline{7})+(1)$	$(21)+(7)$	$(35)+(\overline{7})$
30	(32221110)	$SU_6 \times SU_2 \times U_1^2$	$(15,2)+(\overline{6},1)+(1,1)$	$(\overline{15},1)+(6,2)+(\overline{6},1)$	$(20,1)+(\overline{6},2)+(1,2)$
31	(2222221-1)	$SU_6 \times SU_2 \times U_1^2$	$(20,1)+(\overline{6},2)+(1,2)$	$(15,2)+(\overline{6},1)+(1,1)$	$(\overline{15},1)+(6,2)+(\overline{6},1)$
32	(3222222-1)	$SU_6 \times SU_2 \times U_1^2$	$(\overline{15},1)+(6,2)+(\overline{6},1)$	$(20,1)+(\overline{6},2)+(1,2)$	$(15,2)+(\overline{6},1)+(1,1)$
33	(2222111-1)	$SU_5 \times SU_4 \times U_1$	$(\overline{10},4)$	$(5,6)+(1,\overline{4})$	$(10,1)+(5,4)$
34	(3332211-1)	$SU_5 \times SU_4 \times U_1$	$(10,1)+(5,4)$	$(\overline{10},4)$	$(5,6)+(1,\overline{4})$
35	(3333111-1)	$SU_5 \times SU_4 \times U_1$	$(5,6)+(1,\overline{4})$	$(10,1)+(5,4)$	$(\overline{10},4)$
36	(3322111-1)	$SU_5 \times SU_3 \times SU_2 \times U_1$	$(\overline{5},\overline{3},2)+(5,1,1)$	$(\overline{10},3,1)+(1,\overline{3},2)$	$(\overline{10},1,2)+(5,\overline{3},1)$
37	(33222110)	$SU_5 \times SU_3 \times SU_2 \times U_1$	$(\overline{10},3,1)+(1,\overline{3},2)$	$(\overline{10},1,2)+(5,\overline{3},1)$	$(\overline{5},\overline{3},2)+(5,1,1)$
38	(3322221-1)	$SU_5 \times SU_3 \times SU_2 \times U_1$	$(\overline{10},1,2)+(5,\overline{3},1)$	$(\overline{5},\overline{3},2)+(5,1,1)$	$(\overline{10},3,1)+(1,\overline{3},2)$

Table 7-1. Gauge groups and untwisted matters in Z_8 orbifold models

Superscripts A of shifts denote that gauge groups and matter contents realized in terms of the shifts can be also realized in terms of automorphism. The values of $\sum PJV^J$ include mod 1. Untwisted matters satisfying $\sum P^J V^J = 5/8$, $6/8$ and $7/8$ (mod 1) are antichiral and conjugate (*i.e.*, antiparticles) of those satisfying $\sum P^J V^J = 3/8$, $2/8$ and $1/8$ (mod 1). In Z_8-I, representations in $\sum P^J V^J = 1/8$, $2/8$ and $5/8$ are chiral (or antichiral) and those in $\sum P^J V^J = 4/8$ are projectd out. In Z_8-II, representations in $\sum P^J V^J = 1/8$, $3/8$and $4/8$ are chiral (or antichiral) and those in $\sum P^J V^J = 2/8$ and $6/8$ are projected out.

No.	Shift $8V^J$	Gauge Group $\sum P^J V^J = 0/8$	Untwisted Matters 1/8	2/8	3/8	4/8
0	$(00000000)^A$	E_8				
1	$(22222222)^A$	$E_7 \times SU_2$				(56,2)
2	(11000000)	$E_7 \times U_1$	(56)	(1)		
3	$(11111111)^A$	$E_7 \times U_1$		(56)		2(1)
4	(33000000)	$E_7 \times U_1$		(1)	(56)	
5	(11111100)	$E_6 \times SU_2 \times U_1$	$(\overline{27},2)$	(27,1)	(1,2)	
6	$(22222200)^A$	$E_6 \times SU_2 \times U_1$		$(\overline{27},2)$ +(1,2)		(27,1) $+(\overline{27},1)$
7	(33333300)	$E_6 \times SU_2 \times U_1$	(1,2)	$(\overline{27},1)$	$(\overline{27},2)$	
8	(22111111)	$E_6 \times U_1^2$	$(\overline{27})$+(1)	$(\overline{27})$	(27)+(1)	2(1)
9	(22222211)	$E_6 \times U_1^2$	$(\overline{27})$+(1)	(1)	$(\overline{27})$+(1)	$(27)+(\overline{27})$
10	$(80000000)^A$	SO_{16}				(128_s)
11	(11110000)	$SO_{14} \times U_1$	(64_c)	(14_v)		
12	$(22220000)^A$	$SO_{14} \times U_1$		(64_c)		$2(14_v)$
13	(33330000)	$SO_{14} \times U_1$		(14_v)	(64_c)	
14	(32222221)	$SO_{12} \times SU_2 \times U_1$	$(32_c,1)$	(1,1)	$(12_v,2)$	$(32_s,2)$
15	$(33331111)^A$	$SO_{12} \times SU_2 \times U_1$		$(32_s,1))$ $+(12_v,2)$		$(32_c,2)$ +2(1,1)
16	(44330000)	$SO_{12} \times SU_2 \times U_1$	$(12_v,2)$	(1,1)	$(32_s,1)$	$(32_c,2)$
17	(21111100)	$SO_{12} \times U_1^2$	$(32_c)+(12_v)$	(32_s)+(1)	(12_v)	2(1)
18	$(22221111)^A$	$SO_{12} \times U_1^2$	(32_s)	(12_v)+2(1)	(32_c)	$2(12_v)$
19	(33220000)	$SO_{12} \times U_1^2$	(12_v)	(32_s)+(1)	$(32_c)+(12_v)$	2(1)
20	$(3333311\text{-}1)^A$	$SO_{10} \times SU_4$		$(16_s,4)$		$(10_v,6)$
21	(22111100)	$SO_{10} \times SU_3 \times U_1$	$(16_s,\overline{3})$	$(10_v,3)$	$(16_c,1)$	(1,3) $+(1,\overline{3})$
22	(33332200)	$SO_{10} \times SU_3 \times U_1$	$(16_c,1)$	$(10_v,3)$	$(16_s,3)$	(1,3) $+(1,\overline{3})$
23	$(33222211)^A$	$SO_{10} \times SU_2^2 \times U_1$	$(16_s,2,1)$	$(10_v,1,1)$ +(1,2,2)	$(16_s,1,2)$	$(10_v,2,2)$
24	(22221100)	$SO_{10} \times SU_2 \times U_1^2$	$(16_s,1)$ $+(10_v,2)$	$(16_s,2)$ +(1,1)	$(16_c,1)$ +2(1,2)	$2(10_v,1)$

continued...

Table 7-2. Gauge groups and untwisted matters in Z_8 orbifold models

No.	Shift $8V^J$	Gauge Group $\sum P^J V^J = 0/8$	Untwisted Matters 1/8	2/8	3/8	4/8
25	(22222110)	$SO_{10} \times SU_2 \times U_1^2$	$(16_s, 2)$ +(1,2)	$(16_s, 1)$ $+(10_v, 1)$ +(1,1)	$(10_v, 2)$ +(1,2)	$(16_c, 1)$ $+(16_s, 1)$
26	(33222200)	$SO_{10} \times SU_2 \times U_1^2$	$(16_s, 1)$ +2(1,2)	$(16_s, 2)$ +(1,1)	$(16_s, 1)$ $+(10_v, 2)$	$2(10_v, 1)$
27	(33331100)	$SO_{10} \times SU_2 \times U_1^2$	$(10_v, 2)$ +(1,2)	$(16_c, 1)$ $+(10_v, 1)$ +(1,1)	$(16_s, 2)$ +(1,2)	$(16_c, 1)$ $+(16_s, 1)$
28	$(32222210)^A$	$SO_{10} \times U_1^3$	$(16_c)+(10_v)$ +2(1)	$(16_s)+(10_v)$ +(1)	$(16_s)+(10_v)$ +2(1)	$(16_c)+(16_s)$ +2(1)
29	$(44331100)^A$	$SO_8 \times SU_4 \times U_1$	$(8_c, 4)$	$(8_v, 1)$ +(1,6)	$(8_s, 4)$	$(8_v, 6)$
30	$(3222211\text{-}1)^A$	$SO_8 \times SU_3 \times U_1^2$	$(8_c, \overline{3})$ $+(8_s, 1)$	$(8_v, \overline{3})$ +2(1,3)	$(8_s, 3)$ $+(8_c, 1)$	$2(8_v, 1)$ +(1,3) $+(1, \overline{3})$
31	$(33332110)^A$	$SO_8 \times SU_2^2 \times U_1^2$	$(8_v, 1, 1)$ $+(8_s, 2, 1)$ +(1,2,2)	$(8_c, 2, 1)$ $+(8_s, 1, 2)$ +(1,1,1)	$(8_c, 1, 2)$ $+(8_v, 1, 1)$ +(1,2,2)	$(8_v, 2, 2)$ +2(1,1,1)
32	$(4422222\text{-}2)^A$	$SU_8 \times SU_2$		(28,2)		(70,1)
33	(1111111-1)	$SU_8 \times U_1$	$(\overline{56})$	(28)	$(\overline{8})$	
34	$(2222222\text{-}2)^A$	$SU_8 \times U_1$		$(\overline{56})+(8)$		$(28)+(\overline{28})$
35	(4442111-1)	$SU_8 \times U_1$	$(\overline{8})$	$(\overline{28})$	$(\overline{56})$	
36	(44431000)	$SU_8 \times U_1$	$(\overline{28})$	(1)	$(\overline{28})$	(70)
37	(2211111-1)	$SU_7 \times SU_2 \times U_1$	$(\overline{21}, 2)$	(35,1)	(7,2)	(7,1) $+(\overline{7}, 1)$
38	(3333332-2)	$SU_7 \times SU_2 \times U_1$	$(\overline{7}, 2)$	(35,1)	(21,2)	(7,1) $+(\overline{7}, 1)$
39	(22211110)	$SU_7 \times U_1^2$	$(35)+(\overline{7})$	$(21)+(\overline{7})$	$(\overline{21})+(1)$	$(7)+(\overline{7})$
40	(22222220)	$SU_7 \times U_1^2$	$(\overline{35})+(1)$	2(7)	$(\overline{21})+(7)$	$(21)+(\overline{21})$
41	(3221111-1)	$SU_7 \times U_1^2$	(21)+(7)	(35)	$(\overline{21})+(\overline{7})$	$(7)+(\overline{7})$ +2(1)
42	(33321000)	$SU_7 \times U_1^2$	(21)+(1)	$(21)+(\overline{7})$	$(\overline{35})+(7)$	$(7)+(\overline{7})$
43	(4222222-2)	$SU_7 \times U_1^2$	$(\overline{21})+(7)$	$2(\overline{7})$	$(\overline{35})+(1)$	$(21)+(\overline{21})$

continued...

Table 7-3. Gauge groups and untwisted matters in Z_8 orbifold models

No.	Shift $8V^J$	Gauge Group $\sum P^J V^J = 0/8$	Untwisted Matters 1/8	2/8	3/8	4/8
44	(33222220)	$SU_6 \times SU_2^2 \times U_1$	$(\overline{15},2,1)$	(15,1,1) +(6,1,2)	$(\overline{6},2,2)$ +(1,2,1)	(20,1,2)
45	(4332222-2)	$SU_6 \times SU_2^2 \times U_1$	(6,2,2) +(1,2,1)	(15,1,1) +(6,1,2)	(15,2,1)	(20,1,2)
46	(2222221-1)	$SU_6 \times SU_2 \times U_1^2$	(6,2) +(20,1)	(1,1) +(1,2) +$(\overline{15},2)$	(6,2) +(6,1) +$(\overline{6},1)$	(15,1) +$(\overline{15},1)$
47	(32221110)	$SU_6 \times SU_2 \times U_1^2$	(15,2) +$(\overline{6},1)$	$(\overline{15},1)$ +(6,2) +(1,1)	(20,1) +$(\overline{6},1)$ +(1,2)	(6,2) +$(\overline{6},2)$
48	(3322222-2)	$SU_6 \times SU_2 \times U_1^2$	$(\overline{6},2)$ +(6,1) +$(\overline{6},1)$	$(\overline{15},2)$ +(1,2) +(1,1)	(20,1) +$(\overline{6},2)$	(15,1) +$(\overline{15},1)$
49	(33322210)	$SU_6 \times SU_2 \times U_1^2$	(20,1) +$(\overline{6},1)$ +(1,2)	(15,1) +$(\overline{6},2)$ +(1,1)	(15,2) +$(\overline{6},1)$	(6,2) +$(\overline{6},2)$
50	$(4322222\text{-}1)^A$	$SU_6 \times SU_2 \times U_1^2$	$(\overline{15},1)$ +(6,2) +(1,1)	$(\overline{15},1)$ +$(\overline{6},2)$	(15,1) +$(\overline{6},2)$ +(1,1)	(20,2) +2(1,1)
51	$(3222222\text{-}1)^A$	$SU_6 \times U_1^3$	$(\overline{15})$+(6) +$(\overline{6})$+(1)	(20)+(6) +$(\overline{6})$+(1)	$(\overline{15})$+(6) +$(\overline{6})$+(1)	(15)+$(\overline{15})$ +2(1)
52	(2222111-1)	$SU_5 \times SU_4 \times U_1$	$(\overline{10},\overline{4})$	(5,6)	$(\overline{5},4)$ +(1,4)	(10,1) +$(\overline{10},1)$
53	(3333222-2)	$SU_5 \times SU_4 \times U_1$	(5,4) +(1,4)	(5,6)	$(10,\overline{4})$	(10,1) +$(\overline{10},1)$
54	(3322111-1)	$SU_5 \times SU_3$ $\times SU_2 \times U_1$	$(\overline{5},3,2)$	$(\overline{10},\overline{3},1)$ +(5,1,1)	(10,1,2) +(1,3,2)	(5,3,1) +$(\overline{5},\overline{3},1)$
55	(3333221-1)	$SU_5 \times SU_3$ $\times SU_2 \times U_1$	$(\overline{10},1,2)$ +(1,3,2)	$(\overline{10},3,1)$ +(5,1,1)	(5,3,2)	$(\overline{5},3,1)$ +$(5,\overline{3},1)$
56	(33222110)	$SU_5 \times SU_3 \times U_1^2$	$(10,\overline{3})$ +(1,3) +(1,1)	(10,1) +$(\overline{5},3)$ +(1,3)	(10,1) +$(\overline{5},\overline{3})$ +(5,1)	(5,3) +$(\overline{5},\overline{3})$

continued...

Table 7-4. Gauge groups and untwisted matters in Z_8 orbifold models

No.	Shift $8V^J$	Gauge Group $\sum P^J V^J = 0/8$	Untwisted Matters 1/8	2/8	3/8	4/8
57	$(3332211\text{-}1)^A$	$SU_5 \times SU_3 \times U_1^2$	$(\overline{10},1)$ $+(\overline{5},\overline{3})$ $+(1,3)$	$(10,\overline{3})$ $+(\overline{5},1)$	$(10,1)$ $+(5,3)$ $+(1,\overline{3})$	$(5,\overline{3})$ $+(\overline{5},3)$ $+2(1,1)$
58	(4332221-1)	$SU_5 \times SU_3 \times U_1^2$	$(10,1)$ $+(\overline{5},3)$ $+(5,1)$	$(\overline{10},1)$ $+(5,3)$ $+(1,3)$	$(10,3)$ $+(1,\overline{3})$ $+(1,1)$	$(\overline{5},3)$ $+(5,\overline{3})$
59	$(3322221\text{-}1)^A$	$SU_5 \times SU_2^2 \times U_1$	$(\overline{10},2,1)$ $+(5,1,2)$ $+(1,1,2)$	$(\overline{5},2,2)$ $+2(5,1,1)$	$(10,1,2)$ $+(\overline{5},2,1)$ $+(1,2,1)$	$(10,1,1)$ $+(\overline{10},1,1)$ $+2(1,2,2)$
60	$(4333211\text{-}1)^A$	$SU_4^2 \times SU_2 \times U_1$	$(6,1,2)$ $+(\overline{4},4,1)$	$(1,1,1)$ $+(4,4,2)$	$(\overline{4},4,1)$ $+(1,\overline{6},2)$	$(6,\overline{6},1)$
61	$(3333111\text{-}1)^A$	$SU_4^2 \times U_1^2$	$(6,\overline{4})$ $+(\overline{4},1)$ $+(1,4)$	$(6,1)$ $+(1,6)$ $+(4,\overline{4})$	$(\overline{4},6)$ $+(4,1)$ $+(1,\overline{4})$	$(4,4)$ $+(\overline{4},\overline{4})$
62	$(3332222\text{-}1)^A$	$SU_4 \times SU_3$ $\times SU_2^2 \times U_1$	$(\overline{4},3,2,1)$ $+(4,1,1,2)$	$(6,\overline{3},1,1)$ $+(1,3,2,2)$	$(\overline{4},\overline{3},1,2)$ $+(4,1,2,1)$	$(6,1,2,2)$ $+(1,3,1,1)$ $+(1,\overline{3},1,1)$