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On the Three Triplet Model*

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Abstract It is proved from the substantialistic point of view that difficulties inherent to the quark model can be removed by introducing nine urbaryons which constitute the basis of SU(3)×SU′(3) group. It is shown, also, that the assignment of electric charges to the urbaryons is not unique and a variety of models, including colored quark model and Han-Nambu model, can be accomodated in the scheme presented here.

§ 1 Introduction

The three-triplet model is an old topic but it has become of reviviscent importance in relation to recent investigations on deep inelastic collisions. The three-triplet model was introduced by different authors[1][2][3][4][5] from various motivations. The motivations can be summarized as follows ;

A) the requirement of integral charges.

It is well known that the quarks should have fractional charges. We can assign integral charges to the urbaryons in the three-triplet model. However, the charges of the urbaryons can be integral also in the two-triplet model[6]. Moreover, it is not clear yet, whether the problem of fractional charges is really perplexing or not. I believe that merits of the three-triplet model should be found elsewhere.

B) the problem of statistics.

In the non-relativistic quark model or in the SU(6)×O(3) theory, the three-particle ground state of the urbaryon should correspond to basis of the 56-dimensional representation, so that the state must be symmetric with respect to the space, spin and unitary spin coordinates. This is contradictory with the assumption that the spin of the quarks is one half, as far as the quarks obey the conventional statistics. In order to solve the contradiction, we have only to assume that the quarks obey the parastatistics, or that the quarks have a new degree of freedom, so that the three-particle ground state become antisymmetric with respect to the new degree of freedom. It was shown[7] that

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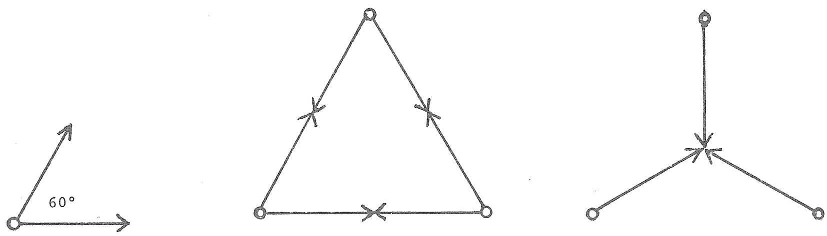
in the case of parastatistics, one is led to the almost same result as the three-triplet model, if one uses Green representation, assuming that a quark is a parafermion of order 3.

C) that the hadrons are non-exotic.

The hadrons which belong to **1**, **8** and **10** dimensional representations of SU(3) group, are observed, while the hadrons belonging to **3**, **6**, **15**, . . . dimensional representations, are missing. In terms of the quark model, three quarks (qqq), a quark and an antiquark ($q\bar{q}$) can be bound, whereas two quarks (qq), two quarks and an antiquark ($qq\bar{q}$) remain unbound.



(a) (b) (c)

Fig. 1

Tati[8] considered that a quark has bonds as shown in Fig. 1 (a), so that only three quarks can be bound as illustrated in Fig. 1 (b). This can be realized mathematically, if one assigns new spin (=1) as a new degree of freedom to the quarks and assumes that a bound state can be formed only when quarks are in an antisymmetric state (singlet state) of the total new spin = 0, (Fig. 1 (c)). This is possible, if one chooses appropriately the new-spin dependence of the quark-quark potential. In terms of the group theory, this means that one chooses the urbaryons to be the basis of the (**3**, **3**) representation of SU(3)×O(3) group. However, antiquarks can not be incorporated into this formalism. It can not be applied to the meson family.

We are able to do almost the same thing with any group other than O(3), if it contains at least one 3-dimensional representation. The most convenient and promising group is SU(3) also for the new degree of freedom. It is possible with SU(3) group as the new degree of freedom to derive the property C for any system which contains antiurbaryons as well as urbaryons. This is the three-triplet model. An urbaryon is assigned to (**3**, **3**) or (**3**, **3***) representation of SU(3)×SU′(3) group.

We shall discuss in more detail the properties A, B and C in the following sections. In the following discussions, we assume the non-relativistic quark model[9], i.e., that the quark-(anti) quark potential is rather flat, so that the nonrelativistic approximation is valid, the kinetic-energy term being negligible. We shall discuss on the validity of this assumption in the last section.

§ 2 The choice of the new degree of freedom

It was mentioned already in the preceding section that the group which characterizes the new degree of freedom, must contain three-dimensional representation.

Let us assume that the quark-(anti) quark potential is described in terms of gluon exchange. Then the group is required to have the property that $\mathbf{3}\times\mathbf{3}$ (or $\mathbf{3}^*\times\mathbf{3}^*$) = sum of two irreducible representations. In the case of Tati theory, for instance, the state which consists of two urbaryons, each having new spin=1, split into the sum of three states with total new spin=0, 1 and 2, respectively, i.e.,

$$\mathbf{3}\times\mathbf{3} = \mathbf{1}+\mathbf{3}+\mathbf{5},$$

where, on the right hand side, $\mathbf{3}$ is antisymmetric, while $\mathbf{1}$ and $\mathbf{5}$ are symmetric. If we confine ourselves to the gluon exchange potential of the second order and choose it to be attractive for the state $\mathbf{3}$, it becomes attractive at the same time either for the $\mathbf{1}$ state or for the $\mathbf{5}$ state.

Finally it should be noted that the urbaryon-antiurbaryon potential $V_{q\bar{q}}$ must be two times more attractive than the urbaryon-urbaryon potential V_{qq}, because

$$meson\ mass = 2M_q + \langle kinetic\ energy\rangle + \langle V_{q\bar{q}}\rangle$$

and

$$baryon\ mass = 3M_q + \langle kinetic\ energy\rangle + 3\langle V_{qq}\rangle,$$

and in the nonrelativistic quark model, meson and baryon masses and the kinetic energy terms can be neglected compared with the quark mass M_q, so that we get

$$\langle V_{q\bar{q}}\rangle \approx -2M_q,$$

and

$$\langle V_{qq}\rangle \approx -M_q. \tag{1}$$

I shall not bother you by tabulating all semi-simple groups, but only show you that the SU(3) group possesses all the required properties.

It is well known that

$$\mathbf{3}\times\mathbf{3} = \mathbf{3}^*+\mathbf{6},\ (or\ \mathbf{3}^*\times\mathbf{3}^* = \mathbf{3}+\mathbf{6}^*)$$

in SU(3). Therefore the first and second requirements are satisfied. We shall assume that the gluon is a vector particle belonging to (1, 8) representation of SU(3)×SU′(3) and denote it as $(B_\mu)^\alpha_\beta$ to derive the third property. The urbaryon can be either (3, 3) or (3, 3*), as was stated in the previous section. Here we take (3, 3) simply for specification and write it as $q_{a\alpha}$, where the latin letters a,b,. . . denote the usual unitary spin indices and the greek letters $\alpha,\beta,\ldots$ stand for the new unitary spin indices. Then the gluon-urbaryon interaction Hamiltonian density is written as,

$$iG \sum_{a\alpha\beta\mu} \bar{q}^{a\alpha}\gamma^{\mu}q_{a\beta}(B_{\mu})^{\alpha}_{\beta}, \tag{2}$$

where G is the super-strong coupling constant. The second-order potentials are derived from this interaction in the non-relativistic approximation as

$$V_{qq} = \sum_{i=1}^{8} \Lambda_i^{(1)}\Lambda_i^{(2)}v(r_{12}), \tag{3}$$

and

$$V_{q\bar{q}} = -\sum_{i=1}^{8} \Lambda_i^{(1)}\Lambda_i^{(2)}v(r_{12}), \tag{4}$$

where Λ_i (i=1,...,8) are the new unitary spin matrices and have the same form as λ_i defined by Gell-Mann. The indices 1 and 2 distinguish two urbaryons (anurbaryon and an anti-urbaryon). r_{12} is the distance between two particles. The minus sign on the left hand side of eq.(4) is due to the vector character of the gluon. As is well known, we have

$$\begin{aligned} <\sum_{i=1}^{8} \Lambda_i^{(1)}\Lambda_i^{(2)}> &= -\frac{8}{3} \textit{ for } \mathbf{3}^* \textit{ antisymmetric qq pair,} \\ &= \frac{4}{3} \textit{ for } \mathbf{6} \textit{ symmetric qq pair,} \\ &= \frac{16}{3} \textit{ for } \mathbf{1}\ q\bar{q} \textit{ pair,} \\ &= -\frac{2}{3} \textit{ for } 8\ q\bar{q} \textit{ pair.} \end{aligned} \tag{5}$$

Therefore, if $v(r) > 0$, the potentials become attractive only for antisymmetric qq pair and for singlet $q\bar{q}$ pair. Eq.(1) is reproduced, if we take

$$<v> \approx \frac{3}{8} M_q. \tag{6}$$

§ 3 The problem of exotic states

Let us consider the mass m of a system which consists of n_q urbaryons and $\bar{n}_q$ antiurbaryons, in the same approximation as in the previous section;

$$m \approx NM_q + <V>, \tag{7}$$

where N is the total number $n_q + \bar{n}_q$ and

$$<V> = <\sum_{l>n=1}^{N} \varepsilon_l\varepsilon_n \sum_{i=1}^{8} \Lambda_i^{(l)}\Lambda^{(n)}{}_i\ v(r_{ln})>, \tag{8}$$

and

$$\begin{aligned} \varepsilon_n &= +1 \textit{ for } n=\textit{urbaryon} \\ &= -1 \textit{ for } n=\textit{antiurbaryon.} \end{aligned}$$

Eq.(8) is readily rewritten as

$$<V> = \frac{1}{2} < \sum_{i=1}^{8} (\sum_{n=1}^{N} \varepsilon_n \Lambda_i^{(n)})^2 - \sum_{n=1}^{N} \sum_{i=1}^{8} (\Lambda_i^{(n)})^2 > <v>. \tag{9}$$

It should be noted here that the new unitary spin of an antiurbaryon is not Λ_i but $-\Lambda_i$, and the total new unitary spin of the system is $\sum_{n=1}^{N} \varepsilon_n \Lambda_i^{(n)}$ instead of $\sum_{n=1}^{N} \Lambda_i^{(n)}$.

An irreducible representation of U(3) group is specified by the 'lengths' (f_1, f_2, f_3) of Yang tableau. An irreducible representation of SU(3) group is specified by (p, q), where $p=f_1-f_2$ and $q=f_2-f_3$. If the total system belongs to the irreducible representation (P, Q), we have

$$< \sum_{i=1}^{8} (\sum_{n=1}^{N} \varepsilon_n \Lambda_i^{(n)})^2 > = \frac{4}{3} (P^2 + PQ + Q^2 + 3(P+Q)), \tag{10}$$

and

$$\begin{aligned} < \sum_{i=1}^{8} (\Lambda_i^{(n)})^2 > &= \frac{4}{3} (p^2 + pq + q^2 + 3(p+q)), \\ &= \frac{16}{3}, \end{aligned} \tag{11}$$

for an urbaryon (or an antiurbaryon), where

$p = 1,\ q = 0$ *for an urbaryon*

and

$p = 0,\ q = 1$ *for an antiurbaryon.*

Therefore the mass of the system is given by

$$m \approx \frac{1}{4} (p^2 + PQ + Q^2 + 3(P+Q)) M_q, \tag{12}$$

where the use has been made of eq.(6). Therefore we conclude that

$m \approx 0$, *only for* $P = Q = 0$,

and

$m \gtrsim M_q$, *otherwise.*

Therefore only the systems which are aggregates of new unitary spin singlet (qqq) and/or $(q\bar{q})$, have low masses compared with M_q. All other systems are too heavy to be observed experimentally. Han and Nambu[2] assumed that the main mass splitting comes from the quadratic Casimir operator of SU′(3). We derived it along the line of thought of the non-relativistic quark model.

§ 4 Electric charges of the urbaryons

It is well known that the Nishijima-Gell-Mann rule is valid for hadrons;

$$Q = I_3 + \frac{1}{2} Y, \tag{13}$$

where Q, I_3 and Y denote the electric charge, the third component of the isospin and the hypercharge, respectively. Because every hadron is new unitary spin singlet as was seen in the previous section, we can modify the N-G-M rule as follows,

$$Q = I_3 + \frac{1}{2}Y + \alpha I_3' + \frac{\beta}{2}Y', \qquad (14)$$

where I_3' and Y' are the third component of the new isospin and the new hypercharge, respectively, α and β being any real numbers. Since the expectation values of I_3' and Y' vanish for a new unitary spin singlet state, the usual N-G-M rule is restored for any system of hadrons. However, the charge of a charmed state (state other than new unitary spin singlet state) is different from that given by the usual N-G-M rule. In particular, the charges of the urbaryons are given by,

$$Q = \frac{2}{3} + \frac{\alpha}{2} + \frac{\beta}{6}, \quad for\ q_{11}$$

$$\cdots\cdot$$

$$= \frac{1}{3} - \frac{\beta}{3}, \quad for\ q_{33}.$$

They are expressed in a more elegant fashion, when expressed in terms of l, m, n, defined by $\alpha = l - m$, $\beta = 2 - 3n$ and $l + m + n = 2$, (Table I). When we choose the urbaryons to be the (3, 3*) representation of SU(3)×SU′(3), we need only to change the signs of α and β.

Table I. The charges of the urbaryons $q_{a\alpha}$, $l+m+n=2$.

a \ α	1	2	3
1	l	m	n
2	$l-1$	m−1	n−1
3	$l-1$	m−1	n−1

Table I remains unchanged.

Similarly we can modify the baryon number as follows,

$$n_B \rightarrow \tilde{n}_B = n_B + \alpha' I_3' + \frac{\beta'}{2}Y'. \qquad (15)$$

Of course one has $\tilde{n}_B = n_B$ for hadrons. On the other hand we can easily obtain, for the urbaryons,

$$\tilde{n} = \frac{1}{3} + \frac{\alpha'}{2} + \frac{\beta'}{6}, \quad for\ q_{a1}$$

$$= \frac{1}{3} - \frac{\alpha'}{2} + \frac{\beta'}{6}, \quad for\ q_{a2}$$

$$= \frac{1}{3} - \frac{\beta'}{3}, \quad for\ q_{a3}$$

where

$$a = 1, 2, 3.$$

Instead of α' and β', we can use n_1, n_2, n_3 defined by $\alpha' = n_1 - n_2$, $\beta' = 1 - 3n_3$ and $n_1 + n_2 + n_3 = 1$, (Table II).

Table II. The modified baryon number $\tilde{n}_3$ of the urbaryons $q_{a\alpha}$, $n_1 + n_2 + n_3 = 1$

a \ α	1	2	3
1, 2, 3	n_1	n_2	n_3

We can combine eqs.(14) and (15) to restore the original N–G–M rule (13). We need not to modify the strangeness S, because it is always integral. Therefore eq.(15) may be regarded as modification of the hypercharge Y,

$$\tilde{Y} = Y + \alpha' I_3' + \frac{\beta'}{2} Y'. \tag{16}$$

If $\alpha' = 2\alpha$ and $\beta' = 2\beta$ or $n_1 = 2l - 1$, $n_2 = 2m - 1$, $n_3 = 2n - 1$, eq.(14) is rewritten in the form,

$$Q = I_3 + \frac{1}{2} \tilde{Y}.$$

The modified hypercharges $\tilde{Y}$ of the urbaryons are tabulated in table III.

Table III. The modified hypercharge $\tilde{Y}$ of the urbaryons, $l + m + n = 2$.

a \ α	1	2	3
1	$2l-1$	$2m-1$	$2n-1$
2	$2l-1$	$2m-1$	$2n-1$
3	$2l-2$	$2m-2$	$2n-2$

As is shown in the above, the choice of the electric charges of the urbaryons is quite arbitrary. Han–Nambu model[2] is only one of examples.

§ 6 Conclusion

We have seen that the three-triplet model successfully solved the difficulties inherent to the quark model. The difficulty of the three-triplet model itself consists in its dynamics.

(i) Are the q–q and q–$\bar{q}$ potentials really flat? It is necessary, because otherwise the expectation value of the kinetic energy would not be negligible compared with the urbaryon mass. It is very likely, however, that the potentials which are derived from the gluon exchange, are singular at the origin as Yukawa potential.

(ii) Is the non-relativistic approximation really good? Of course, to answer the question, the relativistic bound state problem should be solved. It may appear that if the potential is broad and flat, the kinetic energy term is small and the non-relativistic approximation is valid. However, more detailed investigations are necessary, if the potential is so strong that the rest masses are cancelled. Such investigations have been made by Nagasaki[10], Koide-Yamada[11] and Koide[12]. The result is that the non-relativistic approximation is valid only for shallow potentials. The effect of the negative energy sea should be taken seriously into account for such a deep potential as shown in eq.(1). The large mass defect comparable to the rest mass never occurs in the relativistic theory.

In view of recent success of the quark-parton model, it is conjectured that due to the strong attractive potentials V_{qq} and/or $V_{q\bar{q}}$, many unitary spin singlet pairs $(q\bar{q})$ are created and, as the result, the above-mentioned (i) and (ii) are effectively realized. This is analogous to the problem of the relativistic hydrogen-like atom with the atomic number $Z>137$. Such problems still remain unsolved.

Note added: I tried to distinguish a quark in the quark model and an urbaryon in the three-triplet model, but failed, because the boundary is obscure. The latter is regarded as a quark endowed with the new degree of freedom.

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