300°K (0.027eV). These mean free paths are considerably large, compared with the scale of the apparatus. Therefore, we treat the plasma as collision-free hereafter.

Table I. The results of the probe measurements in the case of zero beam (paper2)).
I_A : discharge current T_e : electron temperature
n_e : electron density V_p : plasma potential
f_{pi} : ion plasma frequency.

I_A(mA)	T_e(eV)	n_e(x10^7cm^{-3})	V_p(V)	f_{pi} (KHz calculated)
29	0.93	8.27	0.24	135
33	0.90	8.55	0.14	137
37	0.99	9.25	0.20	142

Table II. The results of the probe measurements in the case of beam injected (paper2)). λ_{De} : Debye length Other notations are the same as Table I. beam current I_b=110μA accelerating voltage V_{acc}=250V.

I_A (mA)	T_e(eV)	n_e(x10^7cm^{-3})	V_p(V)	f_{pi} (KHz calculated)	f_{pi} (KHz observed)	λ_{De}(mm)
29	1.08	7.91	0.80	132	130	0.85
33	1.12	8.49	0.87	136	134	0.83
37	1.12	8.85	0.90	139	136	0.81

Table III. Mean free paths and collision frequencies.
suffix e : electron suffix i : ion
suffix n : neutral molecule.

$\lambda_{e\to n}$=6.7x10^7cm	$\nu_{e\to n}$=6.2x10^5sec^{-1}	
$\lambda_{e\to i}$=7.5x10^3cm	$\nu_{e\to i}$=8.9x10^3sec^{-1}	t_D, Spitzer[5])
$\lambda_{e\to e}$=1.9x10^5cm	$\nu_{e\to e}$=3.5x10^3sec^{-1}	t_c, Spitzer[5])
$\lambda_{i\to n}$=2.1x10 cm	$\nu_{i\to n}$=1.1x10^3sec^{-1}	
$\lambda_{i\to e}$=1.8x10^6cm	$\nu_{i\to e}$=8.9x10^{-3}sec^{-1}	t_s, Spitzer[5])
$\lambda_{i\to i}$=4.0x10 cm	$\nu_{i\to i}$=5.6x10^2sec^{-1}	t_c, Spitzer[5])

3. Fundamental dispersion equation

The grid are grounded and the potential of the surrounding plasma is several volts. Therefore, the ion sheath are formed around the grid. We consider the excitation of an ion plasma oscillation by an electron beam in this ion sheath. Therefore, the system to be considered is the three components system consisting of the ions, the drift electrons, and the beam electrons.

According to the transport theory, the dispersion equation of the multicomponents system in the case of zero magnetic field is

$$1=\sum_j \frac{\omega_{pj}^2}{(\omega-ku_{dj})^2} \tag{1}$$

where u_{dj} and ω_{pj} are the drift velocity and the plasma angular frequency of j paticle, repectively, and

$$\omega_{pj}^2 = \frac{4\pi n_j Z_j^2 e^2}{m_j}, \tag{2}$$

n_j, $Z_j e$ and m_j being the density, the charge, and the mass of j particle, respectively.

Although we tried the numerical calculation, selecting the value of the electron plasma frequency ω_{pe} as the various values equal to or less than the ion plasma frequency ω_{pi}, we could not obtain an appropriate solution for the growth rate depending on a beam current from this cold plasma theory.

Taking into account a finite temperature, one-dimensional dispersion equation is

$$1 = \sum_j \omega_{pj}^2 \int_{-\infty}^{\infty} \frac{f_j(v_j)}{(\omega - kv_j)^2} dv_j \tag{3}$$

$$= -\sum_j \frac{\omega_{pj}^2}{k} \int_{-\infty}^{\infty} \frac{\partial f_j(v_j)}{\partial v_j} \Big/ (\omega - kv_j)\, dv_j. \tag{4}$$

When the velocity distribution of j particle is given by the Maxwell's distribution, the dispersion equation (4) is written as follows, by using the plasma dispresion function.[4)]

$$2k^2 = \sum_j k_{dj}^2 Z'(\zeta_j), \tag{5}$$

$$Z(\zeta) = 2i\exp(-\zeta^2) \int_{-\infty}^{\zeta} \exp(-t^2)\, dt, \tag{6}$$

$$Z'(\zeta) = 2[1 + \zeta Z(\zeta)], \tag{7}$$

$$\zeta_j = \frac{1}{\sqrt{2} V_{Tj}} \left(\frac{\omega}{k} - u_{dj} \right), \quad V_{Tj} = \left(\frac{\kappa T_j}{m_j} \right)^{\frac{1}{2}}, \quad k_{dj}^2 = \frac{4\pi n_j Z_j^2 e^2}{\kappa T_j}, \tag{8}$$

here T_j is the temperature of j particle and κ is Boltzmann's constant.

After Briggs[5)], we simplify the calculation by taking the velocity distributions of the ion and the electron as square distributions.

That is

$$f_j(v_j) = \frac{1}{2V_{Tsj}} \qquad |v_j - u_{dj}| \leqq V_{Tsj}$$
$$= 0 \qquad |v_j - u_{dj}| > V_{Tsj}. \tag{9}$$

In this square distribution, Landau damping is neglected. But, if $\left|\frac{\omega}{k} - u_{dj}\right| \gg V_{Tsj}$ or $\left|\frac{\omega}{k} - u_{dj}\right| \ll V_{Tsj}$, then this distribution gives a good approximation to the Maxwell's distribution. (For the numerical values used in the numerical calculation, $u_{di} = 0$ and $\frac{\omega}{k} > V_{Tsi}$ for the ion and $\left|\frac{\omega}{k} - u_{de}\right| \cong u_{de} > V_{Tse}$ for the electron.) Using this square distribution, dispersion equation is from the equation (3)

$$\frac{\omega_{pi}^2}{(\omega+ku_{di})^2-k^2 V_{Tsi}^2}+\frac{\omega_{pe}^2}{(\omega-ku_{de})^2-k^2 V_{Tse}^2}+\frac{\omega_{pb}^2}{(\omega-ku_b)^2}=1. \tag{10}$$

In spite of that the wave lengths obtained by the numerical calculation are sometimes longer than the diameter of the beam (5mmϕ), the one-dimensional dispersion equation (10) may be used. The reason for this is following. In the case of longitudinal wave, qusistatic method can be applied and furthermore the beam and the plasma are assumed to have the same transverse dimension, because the beam surface effect is considered to be insignificant.[6] In this case with zero magnetic field, the dispersion equation of quasistatic method turns out to be one-dimensional equation.[7]

4. Assumption for numerical calculation and its results

In the numerical calculation of the dispersion equation (10), following numerical values or assumptions are used.

(1) $V_{Tsi}=\frac{1}{\sqrt{2}}V_{Ti}$, $V_{Tse}=V_{Te}$, $T_i=0.04\text{eV}$, $T_e=1.12\text{eV}$

(2) $u_{di}=0$

u_{de} are caculated as follows.

$$I_A=eu_{de}\int_0^{r_0} n(r)\, 2\pi r dr,\quad n(r)=n_0\left\{1-\left(\frac{r}{r_0}\right)^2\right\},$$

where r_0 is the radius of the tube and n_0 is the electron density at the center of the tube. If I_A is taken as 33mA, $\frac{1}{2}m_e u_{de}^2=8.4\text{eV}$ is given from this.

Or, considering u_{de} to be small near the tube wall,

$$I_A=eu_{de}\int_0^{\frac{2}{3}r_0} n(r)\, 2\pi r dr.$$

From this, $\frac{1}{2}m_e u_{de}^2=17.2\text{eV}$ is obtained. These two values of u_{de} are used. 17.2eV may be considered to be rather large. But, if the plasma potential is taken to be 8V as mentioned later, the maximum acceleration voltage is 14V and close to the above value, because the heater voltage is $\pm$6V.

(3) In the ion sheath, the electron density is assumed to obey Boltzmann's law. That is, if the electron densities in the ion sheath and the neutral plasma are n_e and n_0, respectively, then

$$n_e=n_0 e^{-\frac{e(V_p-\phi(x))}{\kappa T_e}}=n_0 e^{-\frac{e\Delta\phi}{\kappa T_e}}, \tag{11}$$

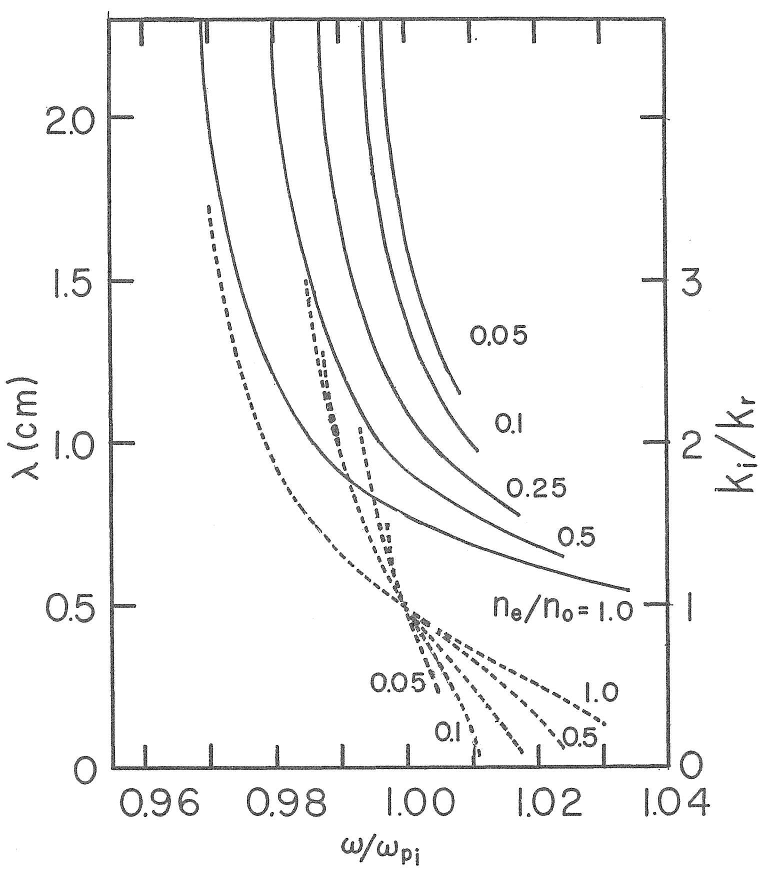
where V_p and $\phi(x)$ are the plasma potential and the potential at the point x, respectively, and $\Delta\phi = V_p - \phi(x)$. But the drift velocity u_{de} is assumed to be constant, independent of the position in the ion sheath and equal to the value at the neutral plasma.

(4) Ion density is assumed to be constant, independent of the position in the ion sheath and equal to the value at the neutral plasma.[8)]

We take ω as real numbers and k as complex numbers and solved the dispersion equation (10) numerically for the various values of n_e/n_0. The wave length λ and growth rate k_i/k_r as the function of ω/ω_{pi} are plotted in Fig. 1, taking n_e/n_0 as the parameter. As ω/ω_{pi} becomes smaller, the wave length becomes longer remarkably and becomes larger than the size of the apparaus. As the solution of such a long wave length is impermissible, we have the solution which centers at $\omega=\omega_{pi}$ and has frequency band width as wide as knwon from Fig. 1. In this band width, for each value of ω there exist two solution of the forward wave and the backward wave having almost the same wave length and growth rate. This band width is considerably wide near $n_e/n_0 = 1.0$, but very narrow at n_e/n_0 below 0.1. Fig. 1. shows the solution in the case of beam currrent $I_b=0$, but the tendency is similar at the presence of the beam cnrrent.

The wave length λ at the central frequencies $\omega=\omega_{pi}$ as the function of n_e/n_0 are

Fig.1 T. Kura and T. Yagi



plotted in Fig. 2 and Fig. 3, taking I_b as parameter. Over the whole range of this figures, the growth rate is constant and equal to about 1.0. In Table IV, the values of n_e/n_0 calculated by the equation (11) and the wave lengths λ for the various values of I_b obtained from Fig. 2 are shown for various $\Delta\phi$ values.

Fig.2 T. Kura and T. Yagi

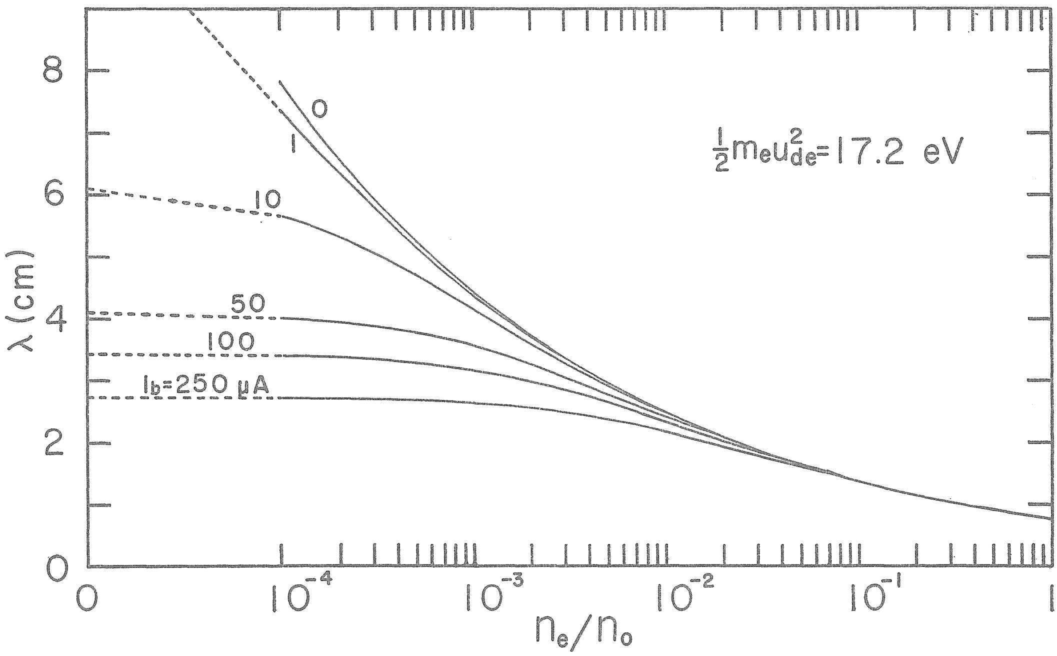


Table IV The relation between the potential $\Delta\phi$, n_e/n_0 and the wave length λ (the case of Fig.2).

$\Delta\phi$(V)	n_e/n_0	λ (cm)				
		I_b=0μA	I_b=10μA	I_b=50μA	I_b=100μA	I_b=250μA
0	1	0.77	0.77	0.77	0.77	0.77
1	0.412	0.96	0.96	0.96	0.96	0.96
2	0.167	1.20	1.20	1.20	1.20	1.20
3	0.0686	1.51	1.51	1.51	1.50	1.49
4	0.0282	1.90	1.90	1.88	1.85	1.80
5	0.0114	2.41	2.41	2.31	2.24	2.12
6	0.0048	2.98	2.92	2.78	2.65	2.39
7	0.0019	3.76	3.60	3.28	3.00	2.61
8	0.00078	4.69	4.33	3.65	3.21	2.65

5. Estimation of damping term

In the dispersion equation (10), the collisional damping and the Landau damping are omitted. Here, we shall estimate them.

Taking $\nu_{in}=1.1\times10^3\text{sec}^{-1}$ and$\omega=\omega_{pi}$, collisional damping rate is given by

$$\frac{k_i}{k_r}=\frac{\nu_{in}}{2\omega}=6.5\times10^{-4},$$

which is negligible.

Next, we shall calculate the Landau damping rate. The electron-beam term is omitted, becaute the beam electrons are assumed not to possess the velocity distribution due to the temperature and the beam velocity $u_b \gg \frac{\omega}{k}$, so that it does not contribute to the Landau damping.

Taking $V_{ph}=\frac{\omega}{k}$, the equation (8) is

$$\zeta_i=\frac{V_{ph}}{\sqrt{2}V_{Ti}},\qquad \zeta_e=\frac{V_{ph}-u_{de}}{\sqrt{2}V_{Te}}\cong-\frac{u_{de}}{\sqrt{2}V_{Te}}$$

$|\zeta_i|>1$, $|\zeta_e|>1$, so that we expand $Z'(\zeta)$.

$$Z'(\zeta)=-2i\pi^{\frac{1}{2}}\zeta\exp(-\zeta^2)+2[\frac{1}{2\zeta^2}+\frac{3}{4\zeta^4}+\cdots\cdots].$$

Substituting this into the equation (5), we otbain

$$k^2=k_{Di}^2[\frac{k^2}{\omega^2}V_{Ti}^2+\cdots]+k_{De}^2[\frac{V_{Te}^2}{u_{de}^2}+\cdots]$$

$$-i\pi^{\frac{1}{2}}k_{Di}^2\frac{V_{ph}}{\sqrt{2}_{Ti}}\exp\left(-\frac{V_{qh}^2}{2V_{Ti}^2}\right)+i\pi^{\frac{1}{2}}k_{De}^2\frac{u_{de}}{\sqrt{2}V_{Te}}\exp\left(-\frac{u_{de}^2}{2V_{Te}^2}\right).$$

If we take $k=k_r+ik_i$, $|k_r|\gg|k_i|$, then we obtain from the real part of this equation

$$\omega^2=\omega_{pi}^2\frac{1}{1-\frac{k_{De}^2}{k^2}\frac{V_{Te}^2}{u_{de}^2}}\qquad(12)$$

This dispersion equation do not coincde with the result of the numerical calculation as shown in Fig.1.

Taking $a=\sqrt{2}V_{Ti}/V_{ph}$ $(a<1)$, we obtain from the imaginary part

$$\frac{k_i}{k_r}=\frac{\pi^{\frac{1}{2}}\left\{\exp(-a^{-2})-\left(\frac{n_e}{n_i}\right)\left(\frac{T_i}{T_e}\right)\frac{u_{de}}{\sqrt{2}V_{Te}}\exp\left(-\frac{u_{de}^2}{2V_{Te}^2}\right)\right\}}{a^3\left\{[1+3a^2+\cdots]-\frac{\omega^2}{\omega_{pi}^2}\right\}}$$

Taking $\omega=\omega_{pi}$ here, we obtain finally

$$\frac{k_i}{k_r}=\frac{\pi^{\frac{1}{2}}}{3a^5}\exp(-a^{-2})-\frac{\pi^{\frac{1}{2}}}{3a^4}\left(\frac{n_e}{n_i}\right)\left(\frac{T_i}{T_e}\right)\frac{u_{de}}{\sqrt{2}\,V_{Te}}\exp\left(-\frac{u_{de}^2}{2V_{Te}}\right). \quad (13)$$

The first term is the Landau damping by ions. The second term is the inverse Landau damping by drift electrons, which is less than 0.1 in the case of $n_e=n_i$. The relation between the ion Landau damping rate and the wave length (i.e. V_{ph}/f_{pi}) is shown in Fig.4. As the wave length is seen longer than 0.77cm from Fig.2, the ion Landau damping is known to be neglible from Fig. 4.

Fig. 3 T. Kura and T. Yagi

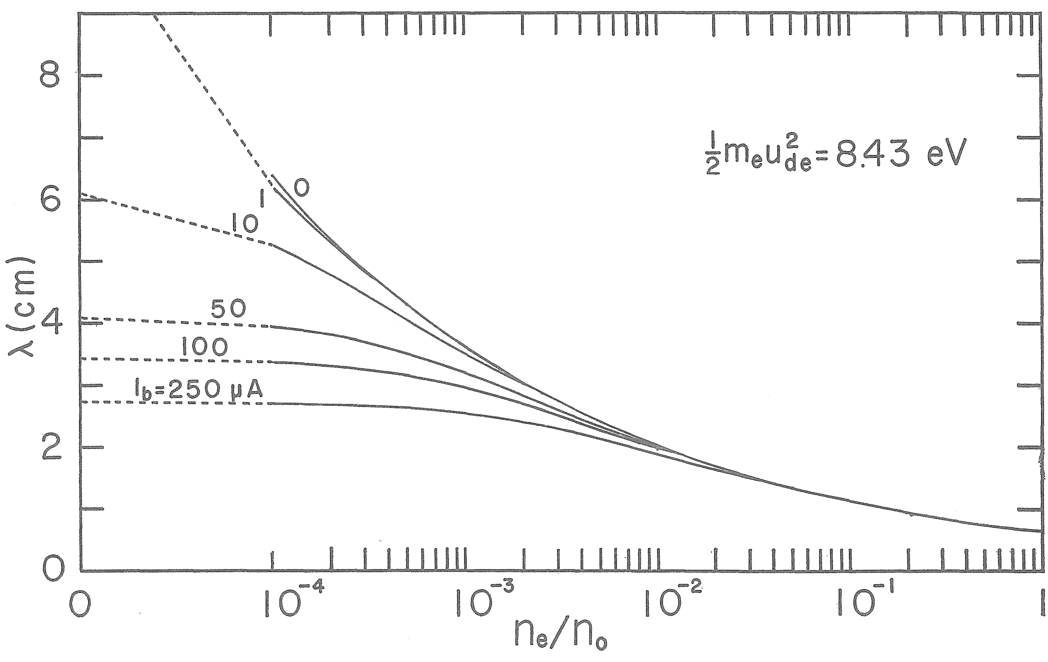
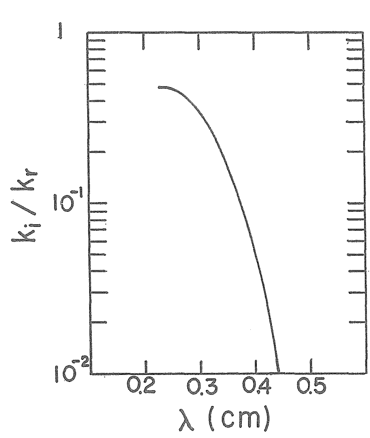


Fig. 4 T. Kura and T. Yagi



6. Excitation mechanism

The solution of the growing wave for the neutral plasma of $n_e/n_0=1$ in Fig. 2 is independent on the beam current and has the broad frequency band width as seen from Fig.1. This is considered to correspond to the spontaneous ion plasma oscillation. As seen in Fig.2 (a) of the paper 1) and Fig.2 (a) of the paper 2), the broad ion plasma oscillations can be recognized when the electron beam is not injected. This spontaneous ion plasma oscillation can be distinguished from the beam excited one by the facts that the former has the broad frequency spectrum and its higher harmonics are extremely small. We leave this solution corresponding to the spontaneous ion plasma oscillation out of consideration.

As seen from Fig.2 and Table IV, in the case of $\Delta\phi=8$V λ is 2.7cm for I_b $=250\mu$A, but 4.7cm for $I_b=0\mu$A which is about two times of the wave length for $I_b=250\mu$A. As there exist the solutions of the forward wave and the backward wave possessing almost the same wave lengths, we consider the standing wave in the ion sheath.[9] The thickness of the ion sheath is about ten times of the Debye length and about 8mm. In the ion sheath n_e/n_0 changes continuously. If the grid is the fixed end and the sheath edge is the free end, the wave length of the standing wave is about 3cm.

Thus, in the case of small I_b, the standing wave can not be excited, because the wave length near the bottom of ion sheath is longer than 3 cm. But, in the case of large I_b, the standing wave can be excited, because the growing wave of the appropriate wave length exists. Furthermore, the frequency band width of the oscillation is very narrow as seen from Fig.1, which agrees with the experimental result. This oscillation is considered to be the beam-excited oscillation. Furthermore, the beam excited ion plasma oscillation whose frequency band width is very narrow may cause the synchronization of the spontaneous ion plasma oscillation whose frequency band width is broad.

A difficult point of this theory is that the plasma potential requires a large value (8V), compared with the experimental results (about 3V in the case of paper 1) and about 1V in the case of paper 2)). However, this value may be accessible, because the measurement of the plasma potential may cause large error owing to the contact potential etc..

7. Summary

In order to investigate the excitation of the ion plasma oscillation due to the mutual interaction between the electron beam and the ion sheath which is produced around the grid placed in the plasma, we carried out the numerical calculation of the dispersion equation of the three components system consisting of the ion of finite temperature, the drift electron of finite temperature, and the beam electron. As the result of the calculation the solution of the growing wave is obtained in the neutral plasma, whose central frequency is ω_{pi} and frequency band width is broad. But this solution is independent on the beam current and considred to correspond to the spontaneous ion plasma oscillation. In the ion sheath, when the beam current is appropriately large, there exists the solution of the growing wave whose central frequency is ω_{pi} and frequency band width is very narrow and the standing wave in the ion sheath can be excited. When the beam current is small, the standing wave can not be excited, because the wave length of the growing wave is too long. The beam excited ion plasma oscillation may cause the synchronization of the spontaneous ion plasma oscillation. In this way, we can explain the experimental result that the ion plasma oscillation is excited by the electron beam, if the beam current is large to some extent. Finally, the numerical calculations were carried out by means of the electronic computor Kanazawa University NEAC-2230.

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In the problem of the pseudo wave, they used the same assumption in considering the motion of the ion in the ion sheath.
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Figure captions

Fig. 1 The result of the numerical calcuation of the dispersion equation. λ and k_i/k_r vs. ω/ω_{pi}. The solid line corresponds to λ, the dotted line to k_i/k_r. $I_b=0$ $\frac{1}{2}m_e u^2{}_{de}=17.2$ eV ($u_{de}=2.44$ x 10^8cm/sec) $n_o=8.49$ x 10^7cm^3 $\omega_{pi}=8.42$ x10^5 rad/sec ($f_{pi}=134$KHz) $\omega_{pe}=5.10$ x 10^8 (n_e/n_o) rad/sec $\omega_{pb}=1.04$ x $10^7\sqrt{I_b}$ rad/sec (I_b in μA) $u_b=9.37$ x 10^8cm/sec ($V_{acc}=250$V) $T_i=0.04$eV $T_e=1.12$eV

Fig. 2 The result of the numerical calculation of the dispersion equation. λ at $\omega=\omega_{pi}$ vs. n_e/n_o. $\frac{1}{2}m_e u_{de}{}^2=17.2$eV ($u_{de}=2.44$ x 10^8cm/sec) Other numerical values are the same as Fig.1.

Fig. 3 The result of the numerical calculation of dispersion equation. λ at $\omega=\omega_{pi}$ vs. n_e/n_o. $\frac{1}{2}m_e u_{de}{}^2=8.43$eV ($u_{de}=1.72$ x 10^8cm/sec) Other numerical values are the same as Fig.1.

Fig. 4 Ion Landau damping rate k_i/k_r vs. λ. $T_i=0.04$eV