# On The Nonlocal Effect in Decay Interactions I

## $\mu - e$ Decay

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## §1. Introduction

Recent experiments on the weak decay processes indicate that (1) nonleptonic decay seems to be understood by the selection rule $|\triangle I| = \frac{1}{2}$ in isobaric space, and (2) leptonic decay seems to be understood according to the universal V-A Fermi interaction.

In the following problems, however, there are some disagreement between the theory and the experiment :

(a) the electron energy spectrum (Michel parameter $\rho$) in the $\mu$ decay[1],
(b) the ratio of $\pi$-$e$ decay to $\pi$-$\mu$ decay[2],
(c) the up-down asymmetry and branching ratio in the $\Lambda$- and $\Sigma$ -decay[3],
(d) the difference of $K_1^0$- and $K^c$-lifetime[4],
(e) the electron (muon) energy spectrum in the $K_{e3}$ ($K_{\mu 3}$) decay[5],
(f) the branching ratio of leptonic to nonleptonic in the hyperon decay[6].

Lee and Yang[7] have shown recently that the $\rho$ value in $\mu$-decay may be explained by a four-fermion intercation taking place over a small space-time region rather than at a single point as is assumed in the usual Fermi theory.

S. Bludman and A. Klein[8] have shown that the analysis of $\mu$-decay must include consideration of certain other logically distinct possibilities in addition to those considered by Lee and Yang.

Furthermore, A. Sirlin[9] have discussed that an effective Hamiltonian is derived which reduces to the local V-A interaction in the limit in which the momentum transfer between the two pairs of Fermi particles is very small, and that for larger values of the momentum transfer corrective terms appear which may account phenomenologically for the deviations of $\rho$ value and the ratio $R = (\pi \rightarrow e + \nu)/(\pi \rightarrow \mu + \nu)$ from the predictions of the local V-A theory.

It is possible that all the Fermi-type interactions do not actually occure with four spinor fields interacting at pricisely the same space-time point. The approach in this paper is to assume the theory in simple and general form.

Phenomenologically, it may be to describe these interactions by a "nonlocal interaction" with these four spinor fields interacting at different space-time points over an extension about $10^{-13} \sim 10^{-14}$ cm, which is the characteristic length of the weak interaction for large momentum transfer processes e. g. $\mu$-decay, $\pi$-decay, and K-decay etc.

The main purpose of this paper is that at the present stage these experiments are analyzed in full generallity, without restrictions to any dynamical model. For the reason, without the reference to any particular form of the theory, we write down directly the most general form of the matrix element for the decay process permissible under the proper Lorentz transformation, assuming the universal V-A theory[10] and the two-component neutrino theory with the lepton conservation[11].

## §2. General formulation

To make our analysis definite, we first consider the case that the $\mu-e$ decay,

$$\mu^- \rightarrow e^- + \nu + \bar{\nu} \tag{1}$$

is represented by a "nonlocal Hamiltonian" of the form

$$H_I = \sum_{i=1}^{5} f_i \iint [\bar{\psi}_e(x)\, O_i\, \psi_\mu(x)]\, F_i(x-x')\, [\bar{\psi}_\nu(x')\, O_i\, \psi_\nu(x')]\, d^4x d^4x', \tag{2}$$

where $f_i$'s are Fermi coupling constants, $\psi_e$, $\psi_\mu$, $\psi_\nu$ are the electron, muon, neutrino field respectively, $\bar{\psi} = \psi^+ \gamma_4$ is adjoint spinor field, $F_i(x-x')$'s are the invariant form factors characterizing the nonlocal interaction, $O_i$'s are $(4 \times 4)$ Dirac matrices i. e.

$$\begin{aligned}
O_1 &= 1 && \text{(scalar)},\\
O_2 &= \gamma_\mu && \text{(vector)},\\
O_3 &= \sigma_{\mu\nu} = \frac{1}{2i}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu) && \text{(tensor)},\\
O_4 &= i\gamma_5 \gamma_\mu && \text{(axial vector)},\\
O_5 &= \gamma_5 && \text{(pseudoscalar)};
\end{aligned} \tag{3}$$

and integrations respect to x and x′ are taken over all space-time.

In eq. (2) the neutrino field satisfies the supplementary condition

$$\gamma_5 \psi_\nu = -\psi_\nu \tag{4}$$

according to the two component neutrino theory. Because of the condition (4), in eq. (2) the S-(scalar), P-(pseudoscalar), and T-(tensor) couplings do not appear. Then index $i$ in eq. (2) sums over only the V-(vector) and A-(axial vector) couplings.

Therefore, in the energy-momentum space, the most general form of the transition matrix for the decay (1) is

$$M = \bar{u}(P)\, \Gamma_\mu\, u(p)\, \bar{u}(q)\, \gamma_\mu \frac{1}{2}(1-\gamma_5)\, v(\bar{q}), \tag{5}$$

where P, p, q, $\bar{q}$ refer to the energy-momenta of muon, electron, neutrino, and antineutrino respectively, and u and v represent the free-particle spinors of positive and negative energy state.

According to the Lorentz invariance the most general four-vector $\Gamma_\mu$ that can be constructed from the available energy momenta and Dirac matrices is

$$\begin{aligned}\Gamma_\mu = (g_0 - g_0'\gamma_5)\gamma_\mu - (i/m)(g_1 - g_1'\gamma_5)p_\mu \\ - (i/m)(g_2 - g_2'\gamma_5)(\gamma_\lambda q_\lambda - \gamma_\lambda \bar{q}_\lambda)\gamma_\mu \\ - (l/m^2)(g_3 - g_3'\gamma_5)(\gamma_\lambda q_\lambda - \gamma_\lambda \bar{q}_\lambda)p_\mu ,\end{aligned} \quad (6)$$

where m is the muon mass. Other invariants, such as $q_\lambda + \bar{q}_\lambda$, do not appear because they are not linearly independent of $q_\lambda - \bar{q}_\lambda$, according to the energy-momentum conservation and Dirac equation.

The $g_i$'s and $g_i''$s in eq. (6) are scalar functions of the following three invariants

$$\begin{aligned} P_1 &= (q + \bar{q})^2/m^2 \sim 2(q.\bar{q})/m^2, \\ P_2 &= (p + \bar{q})^2/m^2 \sim 2(p.\bar{q})/m^2, \\ P_3 &= (p + q)^2/m^2 \sim 2(p.q)/m^2, \end{aligned} \quad (7)$$

constructed from the independent momenta p, q, $\bar{q}$ considered.

Only two of these three invariants in eq. (7) are independent, however, since from the energy-momentum conservation we have

$$1 + P_1 + P_2 + P_3 = O. \quad (8)$$

The original two component neutrino theory leading to $\rho = \frac{3}{4}$ is obtained by omitting the last three terms in eq. (6) and assuming that $g_0$ and $g_0'$ are pure constants independent of $P_i$'s.

There are two kinds of nonlocality.

(a) The first kind nonlocality (the first term in eq. (6)): The three cases in which $g_0$ and $g_0'$ are functions of only one of the $P_i$'s correspond to the simple nonlocal interaction between fermion pairs, each of which interact at the same space-time point. This kind of nonlocality, previously, treated by Lee and Yang, originates where the nonlocality is propagated by virtual bosons or fermion pairs emitted in local energy-momentum independent interactions.

(b) The second kind nonlocality (the last three terms in eq. (6)): These terms in eq. (6) appear phenomenologically as derivative couplings on the electron, neutrino, and electron and neutrino fields respectively, This second nonlocality, treated by Bludman and Klein, can occure where fundamental point (local) interactions are energy-momentum independent, if virtually propagating particles include fermions.

In this paper we treat the effect of the energy-momentum dependence in the coefficients $g_i$'s and $g_i''$s taking account up to not the lowest order but the higher order.

## §3. Calculations and Results

### (I) General case

Our general formula (6) is first approximated by expanding $g_i$'s respect to $P_i$'s up to the first order

$$g_i(P_1, P_2. P_3) = g_i[1 - \varepsilon_1 P_1 - \varepsilon_2 P_2 - \varepsilon_3 P_3],$$
$$g_i'(P_1, P_2, P_3) = g_i'[1 - \varepsilon_1' P_1 - \varepsilon_2' P_2 - \varepsilon_3' P_3],$$
$$(i = 0, 1, 2, 3). \qquad (9)$$

where the $\varepsilon_i$'s and $\varepsilon_i'$'s $(i = 1, 2, 3)$ are small dimensionless parameters measuring the nonlocal effect. If cross terms between derivative couplings and also terms proportional to the electron mass are neglected, the matrix element (5) gives the following electron spectrum of momentum $P = x(m/2)$ emitted into a solid angle $d\Omega$ at an angle $\theta$ with respect to the polarization axis of the parent negative muon $\mu^-$:

$$dN(x, \theta) = \tau (1/3) x^2 dx (4\pi)^{-1} d\Omega (\rho_1 + \rho_2)$$
$$= \tau (1/3) x^2 dx (4\pi)^{-1} d\Omega D \sum_{i=0}^{6} [a_i F_i(x) + \xi \cos\theta\ b_i G_i(x)], \qquad (10)$$

where we take the normalization $\iint dN = 1$ and then $\tau$ is the lifetime of the $\mu^-$. Integrating respect to the electron momentum x from 0 to 1, we have the angular distribution function

$$dN(\theta) = \tau (4\pi)^{-1} d\Omega D \sum_{i=0}^{6} [a_i F_i + \xi \cos\theta\ b_i G_i]. \qquad (11)$$

And then, integrating respect to the angle $\theta$, we obtain the transition probability of the negative muon $\mu^-$

$$\tau^{-1} = (2\pi)^{-3} (m/2)^5 (1/6) D \sum_{i=0}^{6} a_i F_i. \qquad (12)$$

In eq. (10), eq. (11) and eq. (12) various quantities $a_i$'s, $b_i$'s; $F_i(x)$'s, $G_i(x)$'s; $F_i$'s $G_i$'s etc. are given in the appendix.

The spirality of the electron depends on the direction $\theta$ and the momentum x as followes,

$$S = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{\bar{D} \sum_{i=0}^{6} [\bar{a}_i F_i(x) + \bar{\xi} \cos\theta\ \bar{b}_i G_i(x)]}{D \sum_{i=0}^{6} [a_i F_i(x) + \xi \cos\theta\ b_i G_i(x)]}, \qquad (13)$$

where

$$\begin{aligned}\rho_2 = & \mid G_0^2 \mid^2 [F_0(x) - \cos\theta\ G_0(x)] \\ & + \mathrm{Re}\,(G_0^2 G_1^{2*}) [F_1(x) - \cos\theta\ G_1(x)] \\ & + \mid G_1^2 \mid^2 [F_2(x) + \cos\theta\ G_2(x)] \\ & + \mathrm{Re}\,(G_0^2 G_2^{2*}) [F_3(x) - \cos\theta\ G_3(x)] \\ & + \mid G_2^2 \mid^2 [F_4(x) - \cos\theta\ G_4(x)] \\ & + \mathrm{Re}\,(G_0^2 G_3^{2*}) [- F_5(x) + \cos\theta\ G_5(x)] \\ & + \mid G_3^2 \mid^2 [F_6(x) - \cos\theta\ G_6(x)]\end{aligned} \tag{14}$$

is proportional to the square of the matrix element for the emitted electron polarized along it direction of propagation.

The corresponding quantity $\rho_1$ for the electron polarized antiparallel to it direction of propagation is obtained from $\rho_2$ by the substitutions

$$G_0^2,\ G_1^2 \longrightarrow G_0^1,\ G_1^1$$

and (15)

$$G_2^2,\ G_3^2,\quad \cos\theta \longrightarrow -G_2^1,\ -G_3^1,\ -\cos\theta,$$

then

$$\begin{aligned}\rho_1 = & \mid G_0^1 \mid^2 [F_0(x) + \cos\theta\ G_0(x)] \\ & + \mathrm{Re}\,(G_0^1 G_1^{1*}) [F_1(x) + \cos\theta\ G_1(x)] \\ & + \mid G_1^1 \mid^2 [F_2(x) - \cos\theta\ G_2(x)] \\ & + \mathrm{Re}\,(G_0^1 G_2^{1*}) [F_3(x) + \cos\theta\ G_3(x)] \\ & + \mid G_2^1 \mid^2 [F_4(x) + \cos\theta\ G_4(x)] \\ & + \mathrm{Re}\,(G_0^1 G_3^{1*}) [F_5(x) + \cos\theta\ G_5(x)] \\ & + \mid G_3^1 \mid^2 [F_6(x) + \cos\theta\ G_6(x)].\end{aligned} \tag{16}$$

In the numerator of eq. (13), quantities $\bar{D}\bar{a}_i$'s and $\bar{D}\bar{\xi}\bar{b}_i$'s are obtained by the change of a sign of the quantities G's which are attached the subsript 1 in eq. (A6) [see eq (A10)].

The corresponding formula for the positive muon decay are obtained from the above formula by interchanging the subscript 1 and 2 on $G_i^{1,2}$ (i = 1,2,3). This changes the sign of the asymmetry and of the angle-integrated spirality.

### (II) Special Cases : Appropriate approximations.

#### (A) Bludman aod Klein case[8)]

In this case, effective coupling constants $g_i$'s and $g_i'$'s are treated as pure constants, and then $\varepsilon_i$'s and $\varepsilon'_i$'s are taken to be zero. Therfore we have

$$g_i(P_1, P_2, P_3) \equiv g_i,$$

$$g'_i(P_1, P_2, P_3) \equiv g'_i. \tag{17}$$

Then the distribution function leads to the following

$$dN(x, \theta) = \tau\,(1/3)\,x^2\,dx\,(4\pi)^{-1}\,d\Omega D\sum_{i=0}^{6}[a_i\,f_i(x) + \xi\,\cos\theta\;b_i\,g_i(x)], \tag{18}$$

the angular distribution is

$$dN(\theta) = \tau\,(4\pi)^{-1}d\Omega D\sum_{i=0}^{6}[a_i\,f_i + \xi\,\cos\theta\;b_i\,g_i], \tag{19}$$

and the transition probability is

$$\tau^{-1} = (2\pi)^{-3}\,(m/2)^5\,(1/6)D\sum_{i=1}^{6}a_i\,f_i, \tag{20}$$

where

$$f_i = \frac{1}{3}\int_0^1 x^2\,dx\;f_i(x),$$

$$g_i = \frac{1}{3}\int_0^1 x^2\,dx\;g_i(x). \tag{21}$$

#### (B) Lee and Yang case[7)]

If we assume to be $g_0$, $g_0' \neq 0$, and $g_i = g_i' = 0\,(i = 1, 2, 3)$, we have examples discussed by Lee and Yang.

(a) when only $\varepsilon_1$ and $\varepsilon_1'$ are nonvanishing and $\varepsilon_2 = \varepsilon_2' = \varepsilon_3 = \varepsilon_3' = 0$, we have their case I in which $(\nu, \bar{\nu})$ and $(\mu, e)$ interact at different points.

(b) If only $\varepsilon_2$ and $\varepsilon_2'$ are nonzero and $\varepsilon_1 = \varepsilon_1' = \varepsilon_3 = \varepsilon_3' = 0$, then case II is obtained in which $(e, \bar{\nu})$ and $(\mu, \nu)$ interact at different points.

(c) Finally, if we take $\varepsilon_3$ and $\varepsilon_3'$ to be finite and $\varepsilon_1 = \varepsilon_1' = \varepsilon_2 = \varepsilon_2' = 0$, it leads to their case III in which $(e, \nu)$ and $(\mu, \bar{\nu})$ interact at different points.

For example, in order to obtain case I, we put

$$G_0^1,\ G_0^2 \neq 0,\ \text{and}\ G_1^1 = G_2^1 = G_3^1 = G_1^2 = G_2^2 = G_3^2 = 0, \tag{22}$$

and $g_0$ and $g_0'$ are expanded as follows

$$g_0(P_1) = g_0[1 - \varepsilon_1 P_1],$$

$$g_0'(P_1) = g_0'[1 - \varepsilon_1' P_1]. \tag{23}$$

Then the distribution function, the angular distribution, and the total transition probability are given by following formulae :

$$dN(x, \theta) = \tau (1/3) x^2 dx (4\pi)^{-1} d\Omega D [F_0(x) + \xi \cos\theta \ G_0(x)], \tag{24}$$

$$dN(\theta) = \tau (4\pi)^{-1} d\Omega D [F_0 + \xi \cos\theta \ G_0], \tag{25}$$

and

$$\tau^{-1} = (2\pi)^{-3} (m/2)^5 (1/6) DF_0. \tag{26}$$

In these results, eq. (18) – (20) and eq. (24) – (26) coinside with Bludman-Klein's and Lee-Ynag's results respectively. We note that from the above formula we can derive the electron polarization which was discussed previously by Hori, Segawa and Wakasa.[12]

## §4. Discussions of spectrum and asymmetry

From these results we see that the nonlocal effect leads to the small correction with higher momentum dependence. If, according to Bludman and Klein, we take the stand point of second kind nonlocality, various terms of eq. (6) contribute as follows :

(i) Terms $|G_0^{1,2}|^2$ and cross terms between $G_0^{1,2}$ and $G_1^{1,2}$ or $G_2^{1,2}$ contribute linearly in x.

(ii) Cross terms between $G_0^{1,2}$ and $G_3^{1,2}$ and terms $|G_1^{1,2}|^2$ and $|G_2^{1,2}|^2$ contribute quadrically in x.

(iii) Terms $|G_3^{1,2}|^2$ contribute cubically in x.

The most important to be answered by the experiments is whether, aside from the statistical factor $x^2 dx \, d\Omega$, the electron spectrum is linear in x or contains higher momentum dependence. None of the present experiments answer this question definitely. It seems, however, that the electron spectrum is almost linear in x within about 10%. Then this shows that in our phenomenological description the amount of derivative coupling must be small compared to that of nonderivative coupling, and that the nonlocal effect present is also small :

$$g_0, g_0' \gg g_i, g_i' \qquad (i = 1, 2, 3),$$

and (27)

$$1 \gg \varepsilon_i, \varepsilon_i' \qquad (i = 1, 2, 3).$$

If the electron spectrum turns out to be linear in x, aside from the statistical factor, then the interaction structure is very simplified. In this case the nonlocality cannot be of the type considered by Lee and Yang but must be ascribed to a small amount of derivative coupling discussed by the Bludman and Klein. In these circumstances, the electron spectrum is strictly given by the Michel's single parameter form (picking up linear in x) :

$$dN(x, \theta) = \tau (1/3) x^2 dx (4\pi)^{-1} d\Omega D [a_0 f_0(x) + a_1 f_1(x) + a_3 f_3(x) + \xi \cos\theta \{b_0 g_0(x) + b_1 g_1(x) + b_3 g_3(x)\}], \tag{28}$$

or

$$dN(x,\theta) = \tau\, x^2\, dx\, (4\pi)^{-1}\, d\Omega D\,[A(1-x) + B\rho(\frac{4}{3}x - 1)$$
$$+ \xi \cos\theta\, \{A'(1-x) + B'\rho'(\frac{4}{3}x - 1)\}], \qquad (29)$$

where

$$\begin{aligned} A &= 3(2a_0 + a_1 + 2a_3), \\ B\rho &= 3(a_0 + a_1), \\ \xi A' &= 2b_0 - 3b_1 - 2b_3, \\ \xi B'\rho' &= 3(b_0 - b_1). \end{aligned} \qquad (30)$$

The electron spirality is also of the simple form

$$S = \frac{\bar{D}[\bar{A}(1-x) + \bar{B}\bar{\rho}(\frac{4}{3}x - 1) - \bar{\xi}\cos\theta\{\bar{A}'(1-x) + \bar{B}'\bar{\rho}'(\frac{4}{3}x - 1)\}]}{D[A(1-x) + B\rho(\frac{4}{3}x - 1) - \xi\cos\theta\{A'(1-x) + B'\rho'(\frac{4}{3}x - 1)\}]}, \qquad (31)$$

where

$$\begin{aligned} \bar{A} &= 3(2\bar{a}_0 + \bar{a}_1 + 2\bar{a}_3), \\ \bar{B}\bar{\rho} &= 3(\bar{a}_0 + \bar{a}_1), \\ \bar{\xi}\bar{A}' &= 2\bar{b}_0 - 3\bar{b}_1 - 2\bar{b}_3, \\ \bar{\xi}\bar{B}'\bar{\rho}' &= 3(\bar{b}_0 - \bar{b}_1). \end{aligned} \qquad (32)$$

In these expressions constants a's and b's and functions f's and g's are given in the appendix.

**(II) Asymmetry**

The present experiment[13] gives $\xi \simeq -1$, which suggests $|G_0^1|^2 \ll |G_0^2|^2$. If we define following quantities

$$\eta_1 = \mathrm{Re}(G_1^2/G_0^2), \qquad \eta_2 = \mathrm{Re}(G_2^2/G_0^2), \qquad (33)$$

we simplify above formala to

$$\rho = \frac{3}{4}(1 + \frac{1}{2}\eta_1 + \eta_2), \qquad \xi = -(1 - 2\eta_1 + 2\eta_2),$$
$$\rho' = \frac{3}{4}(1 + \frac{1}{2}\eta_1 - \eta_2), \qquad S = 1. \qquad (34)$$

If we make use of the present experimental data

$$\rho = 0.77^{1)}, \qquad \xi = -0.87^{13)}, \qquad (35)$$

we obtain a fit

$$\eta_1 = 0.53, \qquad \eta_2 = 0.47, \tag{36}$$

and prediction

$$\rho' = 0.59. \tag{37}$$

The tentative value of $\eta$ leads to a range of nonlocality

$$r_0 = |\eta| \hbar/mc \simeq 1.0 \times 10^{-13}\ \text{cm}, \tag{38}$$

or an intermediate (virtual) fermion mass of about $400m_e$. This value is to be compared with the corresponding results by Lee-Yang and Bludman-Klein as tabulated in the table.

Table. The range of nonlocality and the mass of an intermediate particle.

| | $r_0$ | m |
|---|---|---|
| Lee-Yang | $0.6\times10^{-13}$cm | $600m_e$ |
| Bludman-Klein | $0.2\times10^{-13}$cm | $2000m_e$ |
| Our result | $1.0\times10^{-13}$cm | $400m_e$ |

## §5. Conclusions and summary

The main conclusion in this paper is that the detailed shape of the electron spectrum, asymmetry, and spirality may distinguish a different kind of interaction structure in muon decay. The validity of these interaction forms is determined by the direct experimental tests. In particular, the important result is that a Michel parameter $\rho$ different from 3/4 can be obtained in this phenomenological theory. Assuming the basic interactions to be local nonderivative couplings (as Bludman-Klein case), the result suggests the presence of intermediate states of fermions of hyperonic mass[8].

Similar analysis can be applied to the another problems stated in §1. In particular, the decay involving a large momentum transfer such as in $K_3$ decay has a characteristic nonlocal effect. These problems shall be treated elsewhere. Finally we note that there is a possibility that we make use of the dispersion technique in analysis of the interaction structure of decay processes.

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## Appendix

In this appendix, we give the various quantities:

$$F_i = \frac{1}{3}\int_0^1 x^2\, dx\; F_i(x), \qquad (i=0,\ 1,\cdots,6) \qquad \text{(A1)}$$
$$G_i = \frac{1}{3}\int_0^1 x^2\, dx\; G_i(x);$$

$$F_i(x) = \{1 + (\alpha_i/a_i)\,\rho(x)\}\, f_i(x), \qquad (i=0,\ 1,\cdots,6) \qquad \text{(A2)}$$
$$G_i(x) = \{1 + (\beta_i/a_i)\,\rho(x)\}\, g_i(x);$$

$$\alpha_i\,\rho(x) = \alpha_{i1}\,\rho_1(x) + \alpha_{i2}\,\rho_2(x) + \alpha_{i3}\,\rho_3(x), \qquad (i=0,\ 1,\cdots,6) \qquad \text{(A3)}$$
$$\beta_i\,\rho(x) = \beta_{i1}\,\rho_1(x) + \beta_{i2}\,\rho_2(x) + \beta_{i3}\,\rho_3(x);$$

$$\rho_1(x) = 1 - \gamma_1 x, \qquad \rho_2(x) = 1 - \gamma_2 x, \qquad \rho_3(x) = 1 - \gamma_3 x, \qquad \text{(A4)}$$

where $\gamma_1$, $\gamma_2$, $\gamma_3$ are independent of x;

$$-D\,\alpha_{01} = |G_0^1|^2\,\bar{E}_{01}^1 + |G_0^2|^2\,\bar{E}_{01}^2, \qquad \text{(A5)}$$
$$-D\xi\,\beta_{01} = |G_0^1|^2\,\bar{E}_{01}^1 - |G_0^2|^2\,\bar{E}_{01}^2,$$

and another $\alpha_{ij}$, $\beta_{ij}$ are similar expresions;

$$\bar{E}_{ij}^1 = E_{ij}^1 + E_{ij}^{1*}, \qquad G_i^1\,E_{ij}^1 = g_i\,\varepsilon_j + g_i'\,\varepsilon_j',$$
$$\bar{E}_{ij}^2 = E_{ij}^2 + E_{ij}^{2*}, \qquad G_i^2\,E_{ij}^2 = g_i\,\varepsilon_j - g_i'\,\varepsilon_j' \qquad \text{(A6)}$$
$$(i=0,\ 1,\ 2,\ 3;\ j=1,\ 2,\ 3);$$

$$Da_0 \equiv D = (|G_0^1|^2 + |G_0^2|^2),$$
$$Da_1 = \mathrm{Re}\,(G_0^1\,G_1^{1*} + G_0^2\,G_1^{2*}),$$
$$Da_2 = (|G_1^1|^2 + |G_1^2|^2),$$
$$Da_3 = -\mathrm{Re}\,(G_0^1\,G_2^{1*} - G_0^2\,G_2^{2*}),$$
$$Da_4 = (|G_2^1|^2 + |G_2^2|^2),$$
$$Da_5 = \mathrm{Re}\,(G_0^1\,G_3^{1*} - G_0^2\,G_3^{2*}),$$
$$Da_6 = (|G_3^1|^2 + |G_3^2|^2),$$
$$D\xi\, b_0 \equiv D\xi = (|G_0^1|^2 - |G_0^2|^2), \qquad \text{(A7)}$$
$$D\xi\, b_1 = \mathrm{Re}\,(G_0^1\,G_1^{1*} - G_0^2\,G_1^{2*}),$$
$$D\xi\, b_2 = -(|G_1^1|^2 - |G_1^2|^2),$$

$$D\xi\, b_3 = -\,\mathrm{Re}\,(G_0^1 G_1^{2*} + G_0^2 G_2^{2*}),$$

$$D\xi\, b_4 = (\,|G_2^1|^2 - |G_2^2|^2),$$

$$D\xi\, b_5 = \mathrm{Re}\,(G_0^1 G_3^{1*} + G_0^2 G_3^{2*}),$$

$$D\xi\, b_6 = (\,|G_3^1|^2 - |G_3^2|^2);$$

$$\begin{aligned}
f_0(x) &= 3 - 2x, & g_0(x) &= 1 - 2x,\\
f_1(x) &= x, & g_1(x) &= x,\\
f_2(x) &= \frac{1}{4}x^2, & g_2(x) &= \frac{1}{4}x^2,\\
f_3(x) &= 6(1 - x), & g_3(x) &= 2(1 - x),\\
f_4(x) &= (3 - x)(1 - x), & g_4(x) &= (1 + x)(1 - x),\\
f_5(x) &= x(1 - x), & g_5(x) &= x(1 - x),\\
f_6(x) &= \frac{1}{20}(5 - 4x)x^2; & g_6(x) &= \frac{1}{20}(3 - 4x)x^2;
\end{aligned} \tag{A8}$$

$$\begin{aligned}
G_0^1 &= g_0 + g_0', & G_0^2 &= g_0 - g_0',\\
G_1^1 &= g_1 + g_1', & G_1^2 &= g_1 - g_1',\\
G_2^1 &= g_2 + g_2', & G_2^2 &= g_2 - g_2',\\
G_3^1 &= g_3 + g_3'; & G_3^2 &= g_3 - g_3';
\end{aligned} \tag{A9}$$

$$\begin{aligned}
\bar{D}\bar{a}_0 \equiv \bar{D} &= (-|G_0^1|^2 + |G_0^2|^2),\\
\bar{D}\bar{a}_1 &= \mathrm{Re}\,(-G_0^1 G_1^{1*} + G_0^2 G_1^{2*}),\\
\bar{D}\bar{a}_2 &= (-|G_1^1|^2 + |G_1^2|^2),\\
\bar{D}\bar{a}_3 &= \mathrm{Re}\,(G_0^1 G_2^{1*} + G_0^2 G_2^{2*}),\\
\bar{D}\bar{a}_4 &= (-|G_2^1|^2 + |G_2^2|^2),\\
\bar{D}\bar{a}_5 &= -\,\mathrm{Re}\,(G_0^1 G_3^{1*} + G_0^2 G_3^{2*}),\\
\bar{D}\bar{a}_6 &= (-|G_3^1|^2 + |G_3^2|^2),\\
\bar{D}\bar{\xi}\,\bar{b}_0 \equiv \bar{D}\bar{\xi} &= -(|G_0^1|^2 + |G_0^2|^2),\\
\bar{D}\bar{\xi}\,\bar{b}_1 &= -\,\mathrm{Re}\,(G_0^1 G_1^{1*} + G_0^2 G_1^{2*}),\\
\bar{D}\bar{\xi}\,\bar{b}_2 &= (\,|G_1^1|^2 + |G_1^2|^2),\\
\bar{D}\bar{\xi}\,\bar{b}_3 &= \mathrm{Re}\,(G_0^1 G_2^{1*} - G_0^2 G_2^{2*}),
\end{aligned} \tag{A10}$$

$$\bar{D}\xi\bar{b}_4 = -(|G_2^1|^2 + |G_2^2|^2),$$

$$\bar{D}\xi\bar{b}_5 = \mathrm{Re}(-G_0^1 G_3^{1*} + G_0^2 G_3^{2*}),$$

$$\bar{D}\xi\bar{b}_6 = -(|G_3^1|^2 + |G_3^2|^2);$$

$$\rho = \frac{3}{4}[1 + \frac{1}{2}\eta_1 + \eta_2 + (\frac{1}{2}\eta_1' - \eta_2')\eta_0 - (\frac{1}{4}\eta_1^2 - \eta_2^2)],$$

$$\xi = -[1 - 2\eta_1 + 2\eta_2 + (\eta_1' - 2\eta_0)\eta_0 + (\eta_1 - 2\eta_2)(\eta_1 - \eta_2)],$$

$$\delta = \frac{3}{4}[1 + \frac{1}{2}\eta_1 - \eta_2 - (\frac{1}{2}\eta_1' + \eta_2')\eta_0 + (\frac{1}{2}\eta_1 - \eta_2)(\frac{3}{2}\eta_1 - \eta_2)], \qquad \text{(A11)}$$

$$S = [1 - 2(\frac{1}{2}\eta_1 - \eta_2)^2 - (\eta_1' + 2\eta_2' + 2\eta_0)\eta_0],$$

$$\tau^{-1} = (2\pi)^{-3}(m/2)^5(1/6)6|G_0^2|^2[1 + \frac{1}{2}\eta_1 + \eta_2 + (\frac{1}{2}\eta_1' - \eta_2' + \eta_0)\eta_0];$$

$$\eta_0 = |G_0^1/G_0^2|,$$

$$\eta_1 = \mathrm{Re}(G_1^{2*}/G_0^2),$$

$$\eta_0\eta_1' = \mathrm{Re}\{(G_0^1/G_0^2)(G_1^{1*}/G_0^2)\}, \qquad \text{(A12)}$$

$$\eta_2 = \mathrm{Re}(G_2^{2*}/G_0^2),$$

$$\eta_0\eta_2' = \mathrm{Re}\{(G_0^1/G_0^2)(G_2^{1*}/G_0^2)\}.$$

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