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On the Nuclear Coulomb Energy

By

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Abstruct

The Coulomb energy difference $\Delta E_c(A)=E_c\left(Z=\frac{A+1}{2},\ N=\frac{A-1}{2}\right)-E_c\left(Z=\frac{A+1}{2},\ N=\frac{A+1}{2}\right)$ are accurately expressed by $\Delta E_c(A)=0.613(A-1)A^{-\frac{1}{3}}MeV$, when the nuclei $\left(Z=\frac{A+1}{2},\ N=\frac{A-1}{2}\right)$ have the closed shell structure for protons. (from $A=3$ to$A=41$).

Introduction

The energy difference between mirror nuclei can be determined fairly accurately from the maximum kinetic energy E_{max} in the β-transition between them. Asuming the charge symmetry of nuclear forces, the binding energy difference of mirror nuclei ΔE_c is attributed to the difference of the nuclear Coulomb energies.[1] Classically this is given by $\Delta E_c=\frac{3}{5}\frac{e^2}{R}(A-1)$ if both of nuclear charge of mirror nuclei are distributed uniformly over a sphere of radius R. The equation

$$\Delta E_c=0.592(A-1)A^{-\frac{1}{3}}MeV \tag{1}$$

which corresponds to $R=1.47\times10^{-13}A^{-\frac{1}{3}}cm$ agrees with observations remarkably and provides evidences for the charge symmetry of nuclear forces and the saturation of nuclear density.[2),3] However, the experimental values deviate from (1) considerably in very small A and deviation are over the experimental errors even in large A.

To explain these deviations more precise calculations have been made. The effects of exchange Coulomb energy were considered[4] by Feenberg and Phillips and the large differences $\Delta E_c(A=11)-\Delta E_c(A=9)$ and $\Delta E_c(A=15)-\Delta E_c(A=13)$ and the small differences $\Delta E_c(A=9)-\Delta E_c(A=7)$ and $\Delta E_c(A=13)-\Delta E_c(A=11)$ were explained qualitatively. Another attempt[3),5] made by Bethe was to correlate the Coulomb energy difference of odd nucleus with binding energy of the least strongly bound particles, and small $\Delta E_c(A=13)$ and $\Delta E_c(A=17)$ and large $\Delta E_c(A=11)$ and $\Delta E_c(A=15)$ were correlated to the strong and weak bindings of the last nucleons.

As the variety and accuracy of the experimental data have increased recently, it seems worth while to consider the problem under the newer data.

The Coulomb energy difference between mirror nuclei

In Fig. 1, the binding energy differences between mirror nuclei calculated from E_{max} (except for $A=5, 9, 25$) taken from the table 2 in the Winther and Kofoed-Hansen's paper[6] are plotted against $(A-1)A^{-\frac{1}{3}}$. For $A=5, 9, 25$ less accurate data are used. The solid line passing through the origin is

$$\Delta E_c = 0.613(A-1)A^{-\frac{1}{3}} MeV. \tag{2}$$

The dotted line represents the equation (1). The table 1 contains the experimental value of ΔE_c obtained from E_{max}, the calculated value from equation (2), their difference

$$\delta = \Delta E_c - 0.613(A-1)A^{-\frac{1}{3}} MeV \tag{3}$$

and the binding energy of last nucleon. "Configuration" in the table gives the configuration which corresponds to the nucleon numbers $(A+1)/2$ according to j–j coupling shell model.[7] In Fig. 2 the Feenberg and Goertzel's plot[8] is replaced by the newer data. The solid curve represents the equation (2) : $2\Delta E_c/(A-1) = 1.226A^{-\frac{1}{3}}$.

Table 1

A	ΔE_c	$0.613(A-1)A^{-\frac{1}{3}}$	δ	binding energy of last nucleon	configuration
3	0.7629±0.0005[10),11)]	0.85	−0.09	6.21[18)]	$(S1/2)^2$
5	0.80[12)]	1.44	−0.64	−0.81[8)]	$P3/2$
7	1.645±0.002[13)]	1.92	−0.27	7.26[12)]	$(P3/2)^2$
9	1.84[14)]	2.36	−0.52	1.67[12)]	$(P3/2)^3$
11	2.758±0.003[13)]	2.76	0.00	11.50[12)]	$(P3/2)^4$
13	3.000±0.005[13)]	3.14	−0.14	4.95[12)]	$P1/2$
15	3.483±0.005[10)]	3.48	0.00	10.83[12)]	$(P1/2)^2$
17	3.545±0.006[15)]	3.83	−0.29	4.14[12)]	$d5/2$
19	4.034±0.005[15)]	4.14	−0.11	10.41[12)]	$(d5/2)^2, (S1/2)^2$
21	4.30±0.03[10)]	4.45	−0.15	6.76[12)]	$(d5/2)^3$ 3/2
23	4.873±0.010[15)]	4.77	+0.10	11.28[18)]	$(d5/2)^4$
25	4.77[16)]	5.04	−0.27	6.82[18)]	$(d5/2)^5$
27	5.28±0.10[17)]	5.32	−0.04	13.07[18)]	$(d5/2)^6$
29	5.40±0.15[10)]	5.61	−0.21	9.18[18)]	$S1/2$
31	5.86±0.12[17)]	5.86	0.00	12.83[18)]	$(S1/2)^2$
33	6.23±0.13[17)]	6.13	+0.10	8.45[19)]	$d3/2$
35	6.2±0.2[10)]	6.39	−0.19	12.75[18)]	$(d3/2)^2$
37	6.37±0.13[17)]	6.63	−0.26	8.78[19)]	$(d3/2)^3$
39	6.93±0.15[17)]	6.88	+0.05	13.13[19)]	$(d3/2)^4$
41	6.7±0.2[10)]	7.16	−0.46	8.41[19)]	$f7/2$

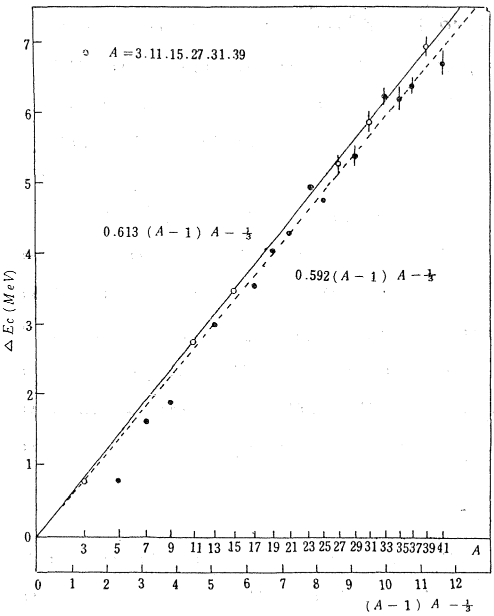

Fig. 1

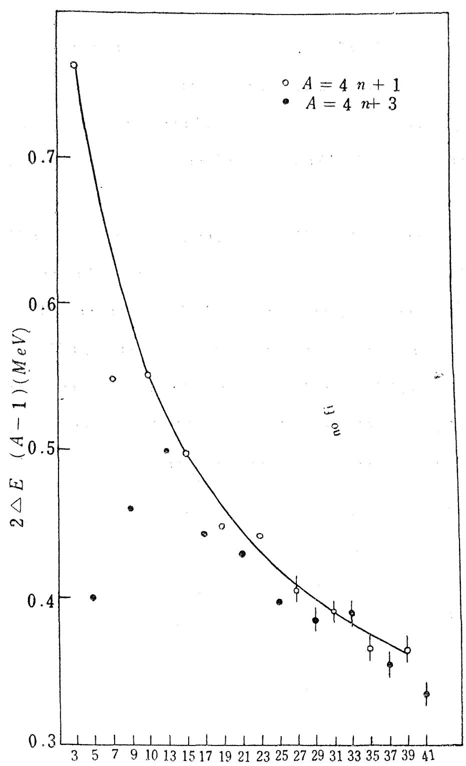

Fig. 2

General features and qualitative interpretations

1. *The effect of exchange energy*. The oscillating behavior of ΔE_c is seen until $A=41$, although it is small for large A, and $\Delta E_c(A=4n+1)$ and $\Delta E_c(A=4n+3)$ lie on relatively smooth curves separately (Fig. 2). This trend was explained by Feenberg and Goertzel[8] as the effect of the exchange Coulomb energy on general assumptions. (Feenberg and Goertzel's theory assumes that the total spin of protons is a good quantum number. According to Kurath[9] the calculated Coulomb energy based on L–S and j–j models are substantially the same.) The exceptions are $A=23$ and $A=33$. The six $\Delta E_c(4n+3)$ connected by the solid line seem to have larger values than other ΔE. This point is discussed in 3.

2. *Correlation with last nucleon biding*. ΔE_c has small or large values according to weak or strong biniding of last nucleon (Table 1). Bethe's theory holds qualitatively until $A=41$. Here again $A=33$ is a exception.

3. *The effect of shell structure*.

(a) ΔE_c for

$$A=3,\ 11,\ 15,\ 27,\ 31,\ 39 \qquad (4)$$

are very closer to the line of equation (2) (Fig. 1, Fig. 2). The deviations δ for these A are smaller than that for other A by the factor about 10 (table 1). (δ for $A=3$ is somewhat larger. The effective radii of nuclei $A=3$ are expected large because of its fewer numbers of nucleons.) Since ΔE_c (A) is the Coulomb energy difference between nuclei $(Z=(A+1)/2,\ N=(A-1/2)$ and $(Z=(A-1)/2,\ N=(A+1)/2)$, and for above six A

$$(A+1)/2=2,\ 6,\ 8,\ 14,\ 16,\ 20, \tag{5}$$

ΔE_c for these A are the Coulomb energy differences between proton closed shell and neutron closed shell nuclei, according to the j–j coupling shellmodel.[7] The proton closed nucleus $(Z=(A+1)/2,\ N=(A-1)/2)$ for the value A of (4) is expected to have a fairly spherical and uniform charge distribution. (Addition theorem for spherical harmonics.) In the neutron closed shell nucleus $(Z=(A-1)/2,\ N=(A+1)/2)$, a fairly spherical and uniform distribution of neutrons may induce the similar proton distribution. (The correlation between proton and neutron distributions are, for instance, can be seen from the ratios of maximum and minimum densities of nucleons due to the Coulomb repulsion between protons. They are estimated $(\rho_{max}/\rho_{min})=1+0.0022A$ for proton and $(\rho_{max}/\rho_{min})1+0.0010A$ for neutron.[20]) Hence, ΔE_c for A in (4) may be expressed as $\Delta E_c=(6/5)Ze^2/R$ fairly well than that for other A. It seems that the fact that ΔE_c for A in (4) is expressed very accurately by the equation (2) is the effect of the nuclear shell structure.

(b) For the value of A larger by two than that in (4), $(Z=(A+1)/2,\ N=(A-1)/2)$ and $(Z=(A-1)/2,\ N=(A+1)/2)$ are neutron and proton closed shell nucleus respectively. These nuclei may also have spherical and uniform charge distributions, but in the closed shell plus one nucleus the last nucleon binding is relatively weak (Table 1) and for these A relatively small ΔE_c is expected from the Beth's theory. This trend appears in observations. (Except $A=33$).

(c) The constant 0.613 in the equation (2) is larger than 0.592 in the equation (1) and δ except for A in (4) is negative. This fact shows that the closed shell nucleus has relatively small effective radius. ΔE_c $(A=23)$ and ΔE_c $(A=33)$ are exceptions and give larger effective radii than those are expected. They are also exceptional cases in the discussions 1 and 2.

(d) Experiments of electron scattering and μ–mesonic X–rays give smaller nuclear radii for the charge distributions of the order of $1.2\times10^{-13}\ A\ cm$.[21][22][23] Measurements with μ–mesonic atoms are accurate in heavior nuclei, while the mirror nuclei experiments are carried out only in lighter nuclei. If the meson experiments give the radii $1.2\times10^{-13}\ A\ cm$ for the lighter nuclei as it seems very likely at the present time, the discrepancy between the values for the nuclear radii derived from mirror nuclei and those derived from meson experiments must be investigated in more detailed calculations.

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